ATOC 5051 INTRODUCTION TO PHYSICAL OCEANOGRAPHY

Lecture 17

Learning objectives: understand and appreciate “mixing processes in the ocean”:

1) Static stability:
   a) Stable and unstable stratification
   b) double diffusion & salt fingering

2) Dynamical instabilities:
   a) Barotropic instability
   b) Baroclinic instability
   c) Kelvin-Helmholtz instability
1) Static stability

- Stable and unstable stratification;

Stable stratification: \( \frac{\partial \rho}{\partial z} < 0 \)

Archimedes Principle states that the buoyant force on a submerged object is equal to the weight of the fluid that is displaced by the object.
Unstable stratification: \[ \frac{\partial \rho}{\partial z} > 0 \]

\[ \rho_a > \rho_w > \rho_b \]
Stable: \( \frac{\partial \rho}{\partial z} < 0 \)

Unstable: \( \frac{\partial \rho}{\partial z} > 0 \)

We often use: buoyancy frequency \( N \) to describe stratification.

\[
N^2 = -\frac{g}{\rho} \frac{\partial \rho}{\partial z}
\]

\( N \): Brunt-Vaisala frequency
• Double diffusion and salt fingering

*Initially - stable*

\[
\frac{\partial \rho}{\partial z} < 0
\]

\[
T_A > T_B, S_A > S_B
\]

T diffusion=100 S diffusion
After certain time:
\[ T_A \approx T_B, S_A > S_B \]
\[ \rho_A > \rho_B, \]

**Unstable**: falling/rising occurs in thin columns

Salt fingering!

Lab experiment: salt fingering

Obs: Mediterranean plume

Small scale mixing: a few meters.
By contrast, if a layer of colder, fresher water is above a layer of warmer, saltier water, the water just above the interface becomes lighter than its above and thus rises, while water below the interface gets heavier and thus sinks.

**Layering:** may result in homogeneous layers Separated by thinner regions of large gradients of T and S.
The ocean is NOT static; they’re in constant motion!
Upper Ocean circulation: long-term mean

Source: Earth's Surface Winds, TERAA Environmental Research Institute

https://www.youtube.com/watch?v=CCmTY0PKGDs

https://vimeo.com/85342340
2) Instabilities: Batotropic, baroclinic, and Kelvin-Helmholtz instabilities

Even though the ocean is stably stratified, motions in the ocean can induce instabilities and thus render the ocean dynamically unstable.

⇒ Perturbation can grow by extracting energy from the background state (large-scale circulation pattern).

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + A_x \frac{\partial^2 v}{\partial x^2} + A_y \frac{\partial^2 v}{\partial y^2} + A_z \frac{\partial^2 v}{\partial z^2}
\]

Eddy mixing

Eddy heat & salt transport
a) Barotropic instability

Horizontal shears of currents are strong: say western boundary region, equatorial current systems.

Equatorial Current Systems: Barotropic Instabilities often Occur

https://twitter.com/mathewabarlow/status/1073372146896912384?lang=en
To isolate barotropic instability, we use a homogeneous ocean with density $\rho_0$ and background current: $U = U(y)$.
The nonlinear equations for the barotropic, inviscid ocean with no forcing are:

\[ u_t + u u_x + v u_y - f v + \frac{1}{\rho_0} p_x = 0, \]

\[ v_t + u v_x + v v_y + f u + \frac{1}{\rho_0} p_y = 0, \]

\[ u_x + v_y = 0. \]

Where

\[ u^T = U(y) + \epsilon u + \epsilon^2 u' + ... \]

Note: \( u_t = \frac{\partial u}{\partial t} \)

Background current

Small parameter

perturbation
**Purpose:** derive an energy equation to show under what conditions a “small perturbation” of current can “grow”.

Substituting $u^T, v^T, p^T$ into the above equations,

$O(\varepsilon^0): \quad U(y) = -\frac{1}{f \rho_0} P_y(y)$

$O(\varepsilon^1): \quad u_t + U u_x - (f - U_y)v + \frac{1}{\rho_0} p_x = 0,$

$v_t + U v_x + fu + \frac{1}{\rho_0} p_y = 0,$

$u_x + v_y = 0.$

$K_E = \frac{1}{2} m(u^2 + v^2);$ for unit mass ($m=1\text{kg/m}^3$),

$K_E = \frac{1}{2} (u^2 + v^2)$
Obtain the kinetic energy’s “time variation” equation for the perturbation and integrate through the channel:

\[
(K_E)_t = \int_{-L_y}^{L_y} \frac{1}{2} (\bar{u}^2 + \bar{v}^2) \, dy = -\int_{-L_y}^{L_y} U_y \bar{u}v \, dy
\]

The overbar represents “average over one wavelength in x direction”.

In order for an instability to grow, \((K_E)_t\) has to be Positive=> Kinetic energy of perturbation increases with time. Thus,

\[
\int_{-L_y}^{L_y} U_y \bar{u}v \, dy < 0
\]
For a perturbation to grow:

\[
\int_{-L_y}^{L_y} U_y \bar{u} v dy < 0 \quad U_y > 0, \bar{u} v < 0
\]

\( a) \) May not grow

\( b) \) Likely grow

(i) Necessary condition for a perturbation to grow, perturbation must tilt against the shear.
Results after the instabilities:

- Barotropic instabilities extract energy from kinetic energy of the background flow.
- Total kinetic energy in the channel is conserved. Instabilities redistribute the energy via mixing effects.
We can derive another equation for **barotropic instability**:

\[ \int_{-L_y}^{L_y} (f - U_y) y \left( \frac{1}{2} \eta^2 \right)_t dy = 0, \]

Where \( v = (\partial_t + U \partial_x) \eta \)

For an instability to grow, \( (\frac{1}{2} \eta^2)_t > 0 \)

Thus,

\[ (f - U_y)_y = \beta - U_{yy} \]

must change sign somewhere in the flow.

**(ii) Necessary condition for barotropic instability:**

**meridional gradient of absolute vorticity**

\[ \beta - U_{yy} = (f + \zeta)_y \] changes sign somewhere in the background flow.
Tropical instability waves (TIWs)

(a) Composite SSHA and Current

Tropical southeast Indian Ocean: Chen et al. 2020, JPO

Tropical Pacific: TIWs

https://www.dailymotion.com/video/xpqjap
TIWs: Tropical Pacific
TIWs: Tropical Atlantic

Sea surface Temperature (SST)

Sea surface Salinity (SSS)

Decco et al. 2018: Ocean dynamics

In this section, the surface energy exchanges of TIWs are discussed. The results shown for the barotropic $\left( \rho_0 \left[ -u^i \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - v^i \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] \right)$, Kelvin-Helmholtz $\left( \rho_0 \left[ -u^i \left( \frac{\partial w}{\partial x} \right) - v^i \left( \frac{\partial w}{\partial y} \right) \right] \right)$ and baroclinic $(-g \rho' \omega')$ energy conversion terms were averaged for the whole reanalysis time span. The reader is referred to the “Appendix” for the kinetic energy equation. The maps of the means (left panels) and standard deviations (right panels) of the three forms of energy conversion are plotted in Fig. 10. The upper maps represent the barotropic (Fig. 10a, b), the middle panels...