Learning objectives: understand the characters (e.g., energy dispersion, solution structure) of the following waves:

1. Coastally trapped waves
2. Equatorial waves:
   (i) Equatorial Kelvin wave;
1. Effects of side boundaries: coastally-trapped waves

Previous classes:
gravity waves &
midlatitude Rossby waves
(surface & internal) - open
ocean;

Today: effects of side
boundaries

Follow the same procedure
as done for open ocean
waves: dispersion relation
& wave solution structure

Coasts act as waveguides.
a) Coastal kelvin wave

Coast. Vertical walls.

\[ f = f_0 > 0 \]
we again use the shallow water equation with \textbf{constant f}: \quad f = f_0 > 0

(Northern Hemisphere)

Find solutions for \( v=0 \), subject to boundary condition: \( v = 0, \; @y = 0 \).

\[
\begin{align*}
v = 0 & \\
\frac{\partial u}{\partial t} &= -g \frac{\partial \eta}{\partial x}, \\
f u &= -g \frac{\partial \eta}{\partial y}, \\
\frac{\partial \eta}{\partial t} + H \frac{\partial u}{\partial x} &= 0.
\end{align*}
\]
From the above equations, we can obtain,

\[ u_{tt} = gH u_{xx} \]

Assume wave form,

\[ u = u(y) e^{i(kx-\omega t)} \]

\[ \omega = \pm kc, \]

where \[ c = \sqrt{gH} \]

This is just like the dispersion relation for long, surface gravity waves in non-rotating system.

*The existence of \( f \) does not affect the dispersion relation!* What’s the effect of \( f \), then?
\[ \eta = \eta(y)e^{i(kx-\omega t)},\ u = u(y)e^{i(kx-\omega t)}, \]

The set of equations yields,

\[ \eta_y = -\frac{f}{k} \eta, \] and choose \( \omega = -kc, \)

\[ \rightarrow \eta_y = +\frac{f}{c} \eta, \]

Thus, \( \eta = \eta_0 e^{+\frac{f}{c} y}e^{i(kx-\omega t)} . \)

Since sea level increases with the increase of \( y, \) which is farther and farther away from the coast, it is not a reasonable solution because energy should decay away from the energy source; for this case the source is the coast.
Choosing $\omega = kc$, we obtain:

$$\eta = \eta_0 e^{-\frac{f}{c} y} e^{i(kx - \omega t)},$$

This solution obtains a maximum at the coast and decays away from it. Reasonable.

When $y = \frac{c}{f}$,

$$\eta = \eta_0 e^{-1} = \frac{\eta_0}{e}$$

$$\approx \eta_0 37.8\%$$
Coastal Kelvin waves: propagate with the coast to its right (left) in Northern (Southern) Hemisphere.

Solutions are trapped to the coast, decaying away from it exponentially, with an e-folding scale of $\frac{c}{f}$, the Rossby radius of deformation.
Satellite Observed Sea Surface Height anomalies: Coastally-trapped waves in the Bay of Bengal of the Indian Ocean (Rao et al. 2010, DSR) (Multi-year mean)

Breakout session: Identify coastal kelvin waves signals
Coastal Kelvin waves: propagate with the coast to its right (left) in Northern (Southern) hemisphere.

Solutions are trapped to the coast, decaying away from it exponentially, with an e-folding scale of $\frac{c}{f}$, the Rossby radius of deformation.
Continental shelf waves: topographic Rossby waves

\[ \text{deep} \]

\[ \text{shallow} \]

\[ \text{Coast} \]
Deep water

H(y)

Shelf
\[
\frac{\partial u}{\partial t} - f v = -g \frac{\partial \eta}{\partial x}, \\
\frac{\partial v}{\partial t} + f u = -g \frac{\partial \eta}{\partial y}, \\
\frac{\partial \eta}{\partial t} + \frac{\partial (H u)}{\partial x} + \frac{\partial (H v)}{\partial y} = 0.
\]

H is depth of water column. Analytic solution, more complicated.

\[
\frac{\partial H}{\partial y}
\]
acts as

\[
\beta = \frac{\partial f}{\partial y}
\]

Topographic Rossby waves
Propagation: like coastal Kelvin waves;

Shelf waves: dispersive.

Mechanism: like Rossby waves: Potential vorticity conservation:

\[ PV = \frac{\zeta + f}{H} = \text{constant} \]

Here, H varies with y because of the shelf; Near the coast, scale is small, \( f \approx \text{constant} \)
Rossby waves

Topographic waves

North

South

\[ D/Dt \left( f + \frac{\zeta}{h} \right) = 0 \]

Waveform motion

NH, North Coast
2. Equatorial waves

(i) The equatorial Kevin wave

a) Dispersion relation:

\[ \omega = kc, \]

barotropic or baroclinic mode speed;

Barotropic mode:

\[ c = c_0 = \sqrt{gH} = 200 \text{m/s} \]

First baroclinic mode:

\[ c_1 = 2 \sim 3 \text{m/s} \]
Phase speed and group velocity of Kelvin waves:

\[ c_p = c_g = c \]

Non-dispersive. Both phase and energy propagate eastward. Exist for all frequencies.
b) Solution:

\[ \eta = e^{-\frac{\beta y^2}{2c}} G'(x - ct) \]
\[ u = \frac{g}{c} e^{-\frac{\beta y^2}{2c}} G'(x - ct) \]
\[ v = 0 \]

E-folding scale: \( a = \sqrt{\frac{2c}{\beta}} \)

Equatorially-trapped: due to \( \beta \)

EQ \( y > 0 \) N.H.

EQ \( y = 0 \)

EQ \( y < 0 \) S.H.

c) Symmetric about the equator (u and p/sea level)

d) **Forcing. Changing winds with time** – symmetric about the EQ.
Equatorial Kelvin wave structure

Red contours: *Symmetric about the equator for Sea surface height (SSH)*
Satellite-observed sea surface height anomalies

Propagation: direction?
Eastward
Westerly wind burst (WWB) associated with the Madden-Julian Oscillation (MJO):
WWB, EQ Kelvin wave & onset of the 1997 El Nino