Chapter 10
Atmospheric Forces & Winds

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What causes the wind to blow?
Atmospheric Pressure

Horizontal pressure variations

How does air pressure change as you move vertically through the atmosphere?

Why does pressure change in this way?

In order to change surface pressure we need to either add or remove air from the column of atmosphere above our point of interest.

For the same surface pressure how will the depth of a column of the atmosphere change as the density of the air in that column changes?

Pressure decreases more rapidly with height in a cold, dense column of air than in a warm, less dense column of air.

The relationship between the change of pressure with height and the density of air is given by the hydrostatic equation:

\[ \frac{\Delta p}{\Delta z} = -\rho g \]

At a given height above the surface the pressure in a warm column of air will be greater than in a cold column of air, assuming both columns of air have the same surface pressure.
How does air respond to a horizontal difference in pressure?

In the figure below how does the surface pressure change in response to the horizontal pressure difference aloft? What causes this change in pressure?

**Pressure gradient force:** A force that acts on air due to spatial differences in pressure.

In response to the change in surface pressure a pressure gradient force is established at the surface that causes air to move from the cold column towards the warm column.

The circulation shown in this figure develops due to differences in heating and cooling of the two columns of air.

Station vs Sea Level Pressure

**Station pressure:** The pressure measured at a particular location

**Calculation:** Use the hydrostatic equation to estimate how much the pressure changes for a 1 km change in elevation.

How does this pressure change compare to horizontal pressure changes seen on a weather map?
Because pressure changes much more quickly in the vertical than in the horizontal direction we need to account for elevation differences when comparing pressure measurements from locations with different elevations.

When station pressure is adjusted to the elevation of sea level it is referred to as **sea level pressure**.

**Isobar**: A line of constant pressure drawn on a map or chart.

By comparing sea level pressure at different locations we can determine the horizontal variation in pressure without worrying about the effect of elevation on the pressure readings.

Sea level pressure can be calculated from the hypsometric equation:

\[ p_2 = p_1 \exp \left( \frac{-g(Z_2 - Z_1)}{RdT} \right) \]

This equation will give sea level pressure \( p_2 \) when:

- \( p_1 \) is the station pressure
- \( z_1 \) is the elevation of the pressure measurement
- \( z_2 = 0 \) m (sea level)
Winds and Weather Maps

**Constant height map:** A weather map that shows how atmospheric pressure at a constant altitude varies with horizontal location.

**Constant pressure (or isobaric) map:** A weather map that shows how the height of a constant pressure surface varies with horizontal location.

If temperature does not vary horizontally and sea level pressure is the same at every location then there will be no change in pressure at any constant height or no change in height at a constant pressure.
How does the height of a constant pressure surface change if the air below it warms or cools?

This variation of height is usually shown with contour lines of constant elevation.

What would the isobars on a constant height map at 5640 m look like for the example to the left?

How do pressure contours on a constant height map compare to height contours on a constant pressure map?

We can compare these two types of contours on a 5 km constant height map and a 500 mb (50 kPa) constant pressure map for the figure at the left.

On a constant pressure map areas of low (high) height correspond to locations of low (high) pressure on a constant height map.
**Ridge:** An area of relatively high heights (or pressure) on a constant pressure (height) map.

**Trough:** An area of relatively low height (pressure) on a constant pressure (height) map.

What is the relationship between wind and contours on a sea level pressure map and a 500 mb constant pressure map?

What are examples of commonly used constant height and constant pressure charts?

| TABLE 8.1 Common Isobaric Charts and Their Approximate Elevation Above Sea Level |
|---------------------------------|-----------------|-----------------|
| ISOBARIC SURFACE (MB) CHARTS     | APPROXIMATE ELEVATION (M) | APPROXIMATE ELEVATION (FT) |
| 1000                            | 120              | 400              |
| 850                             | 1460             | 4800             |
| 700                             | 3000             | 9800             |
| 500                             | 5600             | 18,400           |
| 300                             | 9180             | 30,100           |
| 200                             | 11,800           | 38,700           |
| 100                             | 16,200           | 53,200           |
Newton's 2\textsuperscript{nd} Law

Newton's first law of motion
An object at rest will remain at rest and an object in motion will remain in motion (and travel at a constant velocity along a straight line) as long as no force is exerted on the object.

Newton's second law of motion
The force \((F)\) exerted on an object is equal to mass \((m)\) times the acceleration \((a)\) caused by the force.

\[ F = ma \]

A force can be thought of as a push or pull acting on an object.

Acceleration is the speeding up or slowing down and/or changing of direction of an object and has units of \(\text{m s}^{-2}\).

The acceleration is also the change in velocity of an object over time.

\[ a = \frac{\Delta \vec{V}}{\Delta t} \]

Newton's second law refers to the net force acting on an object.

We can combine the two equations above to give:

\[ \frac{\Delta \vec{V}}{\Delta t} = \frac{F_{\text{net}}}{m} \]

or in component form

\[ \frac{\Delta u}{\Delta t} = \frac{F_{x\text{ net}}}{m}, \frac{\Delta v}{\Delta t} = \frac{F_{y\text{ net}}}{m}, \frac{\Delta w}{\Delta t} = \frac{F_{z\text{ net}}}{m} \]

This equation can be applied to an air parcel in a Lagrangian framework.

What is the velocity of an air parcel that has a zero net force applied to it and how will this change over time?

How does this differ if the net force is non-zero?

These equations allow us to predict how the wind speed and direction will change over time.
What are the horizontal forces that act on air in the atmosphere?

1. Pressure gradient force (PGF or $F_{PG}$)
2. Centrifugal force (CeF or $F_{CN}$)
3. Coriolis force (CoF or $F_{CF}$)
4. Turbulent drag (friction) force (TD or $F_{TD}$)

What is the difference between Lagrangian and Eulerian perspectives?

If we want to apply Newton’s 2nd law in an Eulerian rather than a Lagrangian framework we also need to consider the role of advection in changing the wind ($F_{AD}$).

The textbook treats advection as a force, although it is not a true force.

Taking account of all of these forces Newton’s 2nd law gives the equations of motion for the atmosphere:

\[
\frac{\Delta u}{\Delta t} = \frac{F_{x\ AD}}{m} + \frac{F_{x\ PG}}{m} + \frac{F_{x\ CN}}{m} + \frac{F_{x\ CF}}{m} + \frac{F_{x\ TD}}{m}
\]

\[
\frac{\Delta v}{\Delta t} = \frac{F_{y\ AD}}{m} + \frac{F_{y\ PG}}{m} + \frac{F_{y\ CN}}{m} + \frac{F_{y\ CF}}{m} + \frac{F_{y\ TD}}{m}
\]

Horizontal Forces

Pressure gradient force (PGF or $F_{PG}$)

Why is higher pressure found at the bottom of tank A than at the bottom of tank B?

What force causes the water to flow from tank A to tank B?
Pressure gradient: A measure of the rate at which pressure changes over some distance.

\[ PG = \frac{\Delta p}{\Delta d} \]

What is the pressure gradient between points 1 and 2?

How would this pressure gradient change if the isobars were spaced further apart (closer together)?

The force exerted on the air by the presence of a pressure gradient is known as the pressure gradient force (PGF).

Note that atmospheric scientists typically refer to forces per unit mass and this quantity has units of m s\(^{-2}\).

What is the direction of the PGF relative to the isobars?

How does the magnitude of the PGF change as the spacing of the isobars changes?
The pressure gradient force is the force responsible for causing the wind to blow.

How does the wind speed change as the PGF changes?

The magnitude of the pressure gradient force, on a constant height map, can be expressed as:

$$ F_{PG} = \left| -\frac{1}{\rho} \frac{\Delta p}{\Delta d} \right| $$

where $\rho$ is the density (kg m$^{-3}$), $\Delta p$ is the pressure difference (Pa) on a constant height map, and $\Delta d$ is the distance between pressure contours (m).

On a constant pressure map the magnitude of the PGF can be expressed as:

$$ F_{PG} = \left| -g \frac{\Delta Z}{\Delta d} \right| $$

where $g$ is the acceleration of gravity (=9.8 m s$^{-1}$) and $\Delta Z$ is the difference in height contours (m) on a constant pressure map.
The pressure gradient force in the east/west (zonal) and north/south (meridional) directions can be calculated as:

\[
\frac{F_{x, PG}}{m} = -\frac{1}{\rho} \frac{\Delta p}{\Delta x} \quad \frac{F_{y, PG}}{m} = -\frac{1}{\rho} \frac{\Delta p}{\Delta y}
\]

\[
\frac{F_{x, PG}}{m} = -g \frac{\Delta Z}{\Delta x} \quad \frac{F_{y, PG}}{m} = -g \frac{\Delta Z}{\Delta y}
\]

**Centrifugal Force (CeF or \(F_{CN}\))**

If an air parcel is turning it experiences a centripetal acceleration in the direction it is turning.

In the equations of motion this acceleration is represented by a centrifugal force (\(F_{CN}\)) that points out from the direction of the turn.

The magnitude of the centrifugal force is expressed mathematically as:

\[
\left| \frac{F_{CN}}{m} \right| = \frac{M^2}{R}
\]

where \(M\) is the wind speed and \(R\) is the radius of curvature.

The zonal and meridional components of the centrifugal force are:

\[
\frac{F_{x, CN}}{m} = s \frac{vM}{R} \quad \frac{F_{y, CN}}{m} = -s \frac{uM}{R}
\]

where \(u\) and \(v\) are the zonal and meridional components of the wind.

\[
\text{Table 10-2. To apply centrifugal force to separate Cartesian coordinates, a \((+/-)\) sign factor } s \text{ is required.}
\]

<table>
<thead>
<tr>
<th>Hemisphere</th>
<th>For winds encircling a Low Pressure Center</th>
<th>High Pressure Center</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern</td>
<td>-1</td>
<td>+1</td>
</tr>
<tr>
<td>Northern</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

\(s\) is either +1 or -1 as shown in the table at the left.

Alternately, you can remember that \(s\) is +1 for counterclockwise turning and -1 for clockwise turning of the wind.
Coriolis Force (CoF or $F_{CF}$)

**Coriolis force:** An apparent force that arises due to the rotation of Earth

The Coriolis force acts to the right of the wind in the Northern Hemisphere and to the left of the wind in the Southern Hemisphere.

The magnitude of the Coriolis force is:

- proportional to the wind speed
- greater at higher latitudes and zero at the equator

Because the Coriolis force is always perpendicular to the wind it can only change the wind direction and cannot change the wind speed.

The magnitude of the Coriolis force can be expressed as:

$$\left| \frac{F_{CF}}{m} \right| = |fM|$$

where $M$ is the wind speed ($\text{m s}^{-1}$) and $f$ is the Coriolis parameter given by:

$$f = 2\Omega \sin \phi \ (\text{s}^{-1})$$

$\Omega$ is the rotation rate of the planet ($=7.292 \times 10^{-5} \ \text{s}^{-1}$) and $\phi$ is the latitude (positive for Northern Hemisphere, negative for Southern Hemisphere).

The zonal and meridional components of the Coriolis force are given by:

$$\frac{F_{x CF}}{m} = fv$$
$$\frac{F_{y CF}}{m} = -fu$$

What is the sign of $u$ for an east (west) wind? What is the sign and direction of the Coriolis force for an east (west) wind?

What is the sign of $v$ for a north (south) wind? What is the sign and direction of the Coriolis force for a north (south) wind?
Turbulent Drag (Friction) Force (TD or $F_{TD}$)

The turbulent drag (friction) force is a result of turbulence transporting atmospheric momentum to the ground and acts in the atmospheric boundary layer.

This means that the turbulent drag force slows the wind and thus acts in a direction opposite the wind.

The magnitude of the turbulent drag force is given by:

$$\left| \frac{F_{TD}}{m} \right| = C_D M$$

where $C_D$ is a drag coefficient that varies with the intensity of turbulence.

The zonal and meridional components of the turbulent drag force are given by:

$$\frac{F_{x TD}}{m} = -C_D u$$

$$\frac{F_{y TD}}{m} = -C_D v$$

Advection ($F_{AD}$)

The wind can transport air that has a different velocity past a fixed location causing the wind at that fixed location to change over time.

This process of changing the wind at this fixed location is called wind advection.

What factors will determine how quickly advection will change the wind at a fixed location?
Wind from any direction can transport air with a new velocity \((u, v, \text{ or } w)\) past a fixed point.

The advective “force” can be expressed as:

\[
\frac{F_{x, AD}}{m} = -u \frac{\Delta u}{\Delta x} - v \frac{\Delta u}{\Delta y} - w \frac{\Delta u}{\Delta z}
\]

\[
\frac{F_{y, AD}}{m} = -u \frac{\Delta v}{\Delta x} - v \frac{\Delta v}{\Delta y} - w \frac{\Delta v}{\Delta z}
\]

Normally the last term on the RHS of these equations is small and can be neglected.

Of the five forces discussed above (PGF, CoF, CeF, TD, and AD) only the PGF can cause air that is initially at rest to begin moving.

All of the other forces begin to act on the air once the air is in motion.

**Equations of Horizontal Motion**

Considering all of the forces discussed above the horizontal equations of motion, based on Newton’s 2\(^\text{nd}\) law, are:

\[
\frac{\Delta u}{\Delta t} = -u \frac{\Delta u}{\Delta x} - v \frac{\Delta u}{\Delta y} - w \frac{\Delta u}{\Delta z} - \frac{1}{\rho} \frac{\Delta p}{\Delta x} + f v - C_D u
\]

\[
\frac{\Delta v}{\Delta t} = -u \frac{\Delta v}{\Delta x} - v \frac{\Delta v}{\Delta y} - w \frac{\Delta v}{\Delta z} - \frac{1}{\rho} \frac{\Delta p}{\Delta y} - f u - C_D v
\]

What process does each term in these equations represent?
Horizontal Winds

Geostrophic Wind

Consider an air parcel placed within the pressure gradient shown to the left. This air parcel is above the effects of friction from the surface of the earth (altitude > 1 km).

How does this air parcel respond to the pressure gradient force acting on it?

What other force acts on the air parcel once it begins to move?

How does this force change with time?

What is the net force acting on this air parcel at point 5 in the figure above?

At point 5 the flow is steady state and experiences no further acceleration and the PGF is exactly balanced by the CoF.

Geostrophic wind: A straight-line flow of air that is parallel to isobars (or height contours) in which the Coriolis force exactly balances the pressure gradient force.

How does the geostrophic wind speed change as the spacing of isobars (or height contours) decreases?
The geostrophic wind speed \( V_g \) or its zonal \( u_g \) and meridional \( v_g \) components can be calculated from a constant height map using:

\[
V_g = \frac{1}{\rho f} \frac{\Delta p}{\Delta d} \\
u_g = -\frac{1}{\rho f} \frac{\Delta p}{\Delta y} \\
v_g = \frac{1}{\rho f} \frac{\Delta p}{\Delta x}
\]

For a constant pressure map \( V_g, u_g, \) and \( v_g \) can be calculated using:

\[
V_g = -\frac{g}{f} \frac{\Delta Z}{\Delta d} \\
u_g = -\frac{g}{f} \frac{\Delta Z}{\Delta y} \\
v_g = \frac{g}{f} \frac{\Delta Z}{\Delta x}
\]

When calculating the zonal component of the geostrophic wind calculate \( \Delta Z \) as the northern height contour minus the southern height contour.

When calculating the meridional component of the geostrophic wind calculate \( \Delta Z \) as the eastern height contour minus the western height contour.

For all of the equations listed above \( \Delta d, \Delta x, \) and \( \Delta y \) are always positive.

**Gradient Wind**

The geostrophic wind described above is only appropriate if the isobars or height contours are straight.

When these contours are curved then the centrifugal force (CeF) also needs to be considered.
Gradient wind: A wind that blows at constant speed parallel to curved isobars (or height contours) above the level of friction.

For the gradient wind the PGF and CoF due not balance. The net force that results from these two forces is what causes the wind to turn CCW around lows and CW around highs in the Northern Hemisphere and is called the centripetal acceleration.

If we consider the CeF to be the negative of the centripetal acceleration then for the gradient wind the PGF, CoF, and CeF balance.

How does the gradient wind speed around low and high pressure centers compare if both pressure centers have the same magnitude pressure gradient?

How does the gradient wind speed compare to the geostrophic wind speed around a low (high) pressure center?

What does this difference between gradient and geostrophic wind speed imply about the magnitude of the Coriolis force relative to the magnitude of the pressure gradient force for a low (high) pressure center?

The gradient wind speed ($V_{\text{grad}}$) for flow around a low pressure system can be calculated from a constant height (left) or constant pressure (right) map using:

$$V_{\text{grad}} = -\frac{fR}{2} + \left( \frac{f^2 R^2}{4} + \frac{R \Delta p}{\rho \Delta d} \right)^{0.5}$$

The gradient wind speed ($V_{\text{grad}}$) for flow around a high pressure system can be calculated from a constant height (left) or constant pressure (right) map using:

$$V_{\text{grad}} = \frac{fR}{2} - \left( \frac{f^2 R^2}{4} - \frac{R \Delta p}{\rho \Delta d} \right)^{0.5}$$

$$V_{\text{grad}} = \frac{fR}{2} - \left( \frac{f^2 R^2}{4} - \frac{R \Delta Z}{\Delta d} \right)^{0.5}$$

where $R$ is the radius of curvature of the flow.
Winds on upper level charts

Where are the winds on this map in geostrophic (gradient) wind balance?

Atmospheric Boundary Layer Wind

As discussed above the turbulent drag (friction) force acts to slow the air in the atmospheric boundary layer.

How would wind in geostrophic balance respond to the addition of the friction force?
The figures below illustrate the difference between winds aloft, without friction, and those near the ground, under the influence of friction, for straight and curved flow.

Winds being influenced by friction will be:

- slower than the geostrophic (or gradient) wind
- will blow at angle across isobars from high pressure towards low pressure

As a result of friction winds spiral in towards low pressure centers and out from high pressure centers.

How will the magnitude of the friction force differ over a rough or smooth surface?

What impact will this change in friction force have on the wind direction relative to the isobars?

**Vertical Forces and Motion**

What forces act on air in the vertical direction?

\[
\frac{\Delta w}{\Delta t} = -u \frac{\Delta w}{\Delta x} - v \frac{\Delta w}{\Delta y} - w \frac{\Delta w}{\Delta z} - \frac{1}{\rho} \frac{\Delta p}{\Delta z} - |g| - \frac{F_{z,TD}}{m}
\]

The vertical pressure gradient force and gravity are much larger than all of the other forces that act in the vertical direction.
Hydrostatic balance: A balance between the upward directed pressure gradient force and the downward directed gravity force.

This balance is expressed mathematically by the hydrostatic equation:

$$\frac{\Delta p}{\Delta z} = -\rho g$$

Conservation of Mass

Continuity Equation

For a volume of air at a fixed location (Eulerian framework) the difference of the mass of air going into and out of the volume will change the mass, and thus density, of the volume.

The mass flowing into (or out of) each side of the box is given by $\rho u$, $\rho v$, and $\rho w$.

This relationship between mass inflow and outflow and changes in density is expressed by the continuity equation and represents conservation of mass.

$$\frac{\Delta \rho}{\Delta t} = \left[ \frac{\Delta (\rho u)}{\Delta x} + \frac{\Delta (\rho v)}{\Delta y} + \frac{\Delta (\rho w)}{\Delta z} \right]$$

The continuity equation can be re-written as:

$$\frac{\Delta \rho}{\Delta t} = \left\{ -u \frac{\Delta \rho}{\Delta x} - v \frac{\Delta \rho}{\Delta y} - w \frac{\Delta \rho}{\Delta z} \right\} - \rho \left[ \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} + \frac{\Delta w}{\Delta z} \right]$$

What processes do the terms in the curly and square brackets represent?
Vertical Motion

If we neglect variations in density the continuity equation reduces to:

\[ 0 = \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} + \frac{\Delta w}{\Delta z} \]

or

\[ \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} = -\frac{\Delta w}{\Delta z} \]

The equation above on the right indicates that horizontal divergence (or convergence) is balanced by vertical convergence (or divergence).

As a result of friction air converges into low pressure at the surface and diverges from a high pressure at the surface and results in rising motion above low pressure centers and sinking motion above high pressure centers.
How will vertical motion differ for convergence (divergence) at the surface and aloft?

The total convergence (or divergence) in a column of the atmosphere will change the mass of the column and thus the surface pressure.

How will convergence (divergence) alter the surface pressure?

In the figure on the previous page what determines whether the surface pressure will increase, decrease, or remain constant?