



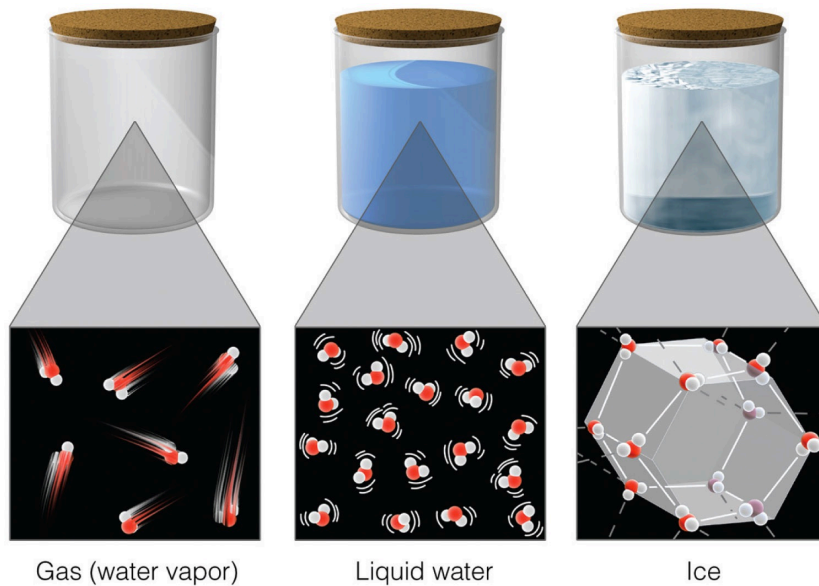
## Chapter 4 Water Vapor

### Chapter overview:

- Phases of water
- Vapor pressure at saturation
- Moisture variables
  - Mixing ratio, specific humidity, relative humidity, dew point temperature
  - Absolute vs. relative humidity
  - Total water
- Moisture and Thermodynamic Diagrams
  - Lagrangian budgets
  - Heat conservation for saturated air
  - Wet bulb and equivalent potential temperature
- Water budget at a fixed location

Why is water in the atmosphere important?

## Phases of Water

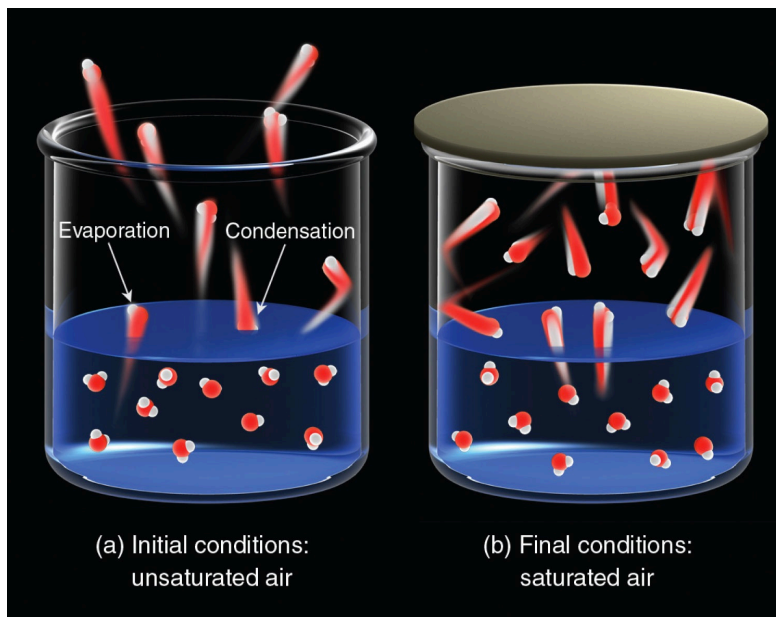


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**Vapor pressure (e):** The pressure exerted by the water vapor molecules in air. (Units: mb)

The vapor pressure increases as the amount of water vapor in the air increases.

## Vapor Pressure at Saturation



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### Evaporation:

Transformation of water from the liquid to the vapor state.

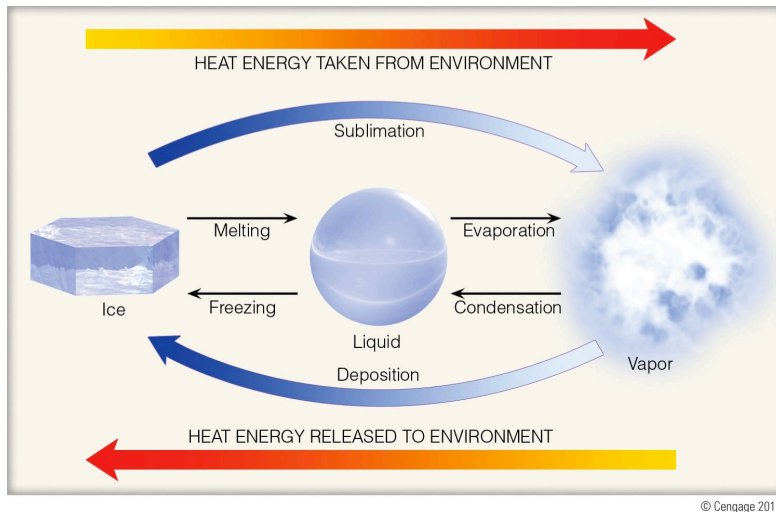
### Condensation:

Transformation of water from the vapor to the liquid state.

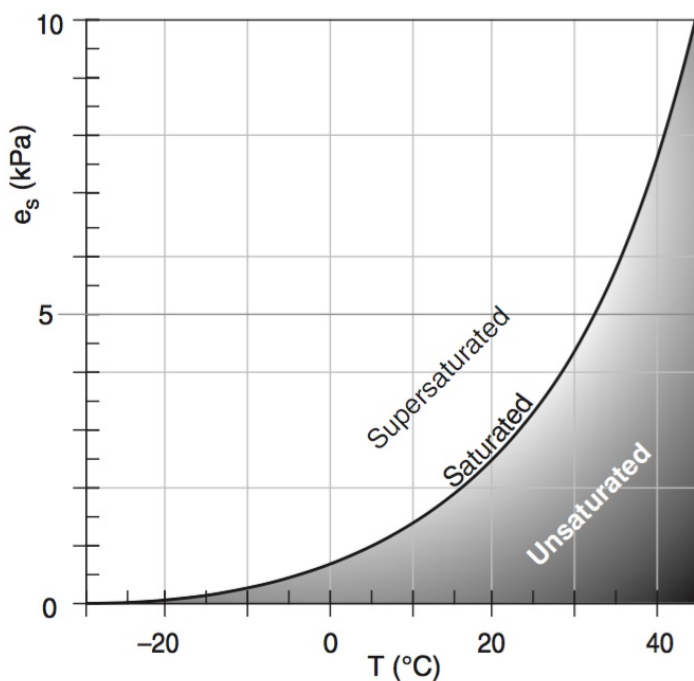
What happens on a molecular scale as water evaporates or condenses?

**Saturation:** A condition in which the number of water molecules evaporating from a liquid surface is exactly balanced by the number of water molecules that are condensing.

What factors influence the rate at which water evaporates / condenses?



Remember: As water changes phase energy is either released to or taken from the surrounding air.



**Saturation vapor pressure ( $e_s$ ):** The pressure that would be exerted by water vapor molecules if the air were saturated with vapor at a given temperature. (Units: mb)

How does saturation vapor pressure change as temperature changes?

Why does this relationship between saturation vapor pressure and temperature exist?

How does the vapor pressure ( $e$ ) compare to the saturation vapor pressure ( $e_s$ ) for unsaturated, saturated, and supersaturated conditions?

How does the atmosphere respond to supersaturated conditions?

The relationship between saturation vapor pressure and temperature is given by the [Clausius-Clapeyron](#) equation:

$$e_s = e_0 \exp \left[ \frac{L}{R_v} \left( \frac{1}{T_0} - \frac{1}{T} \right) \right] \text{ (units: mb)}$$

$$e_0 = 6.113 \text{ mb}$$

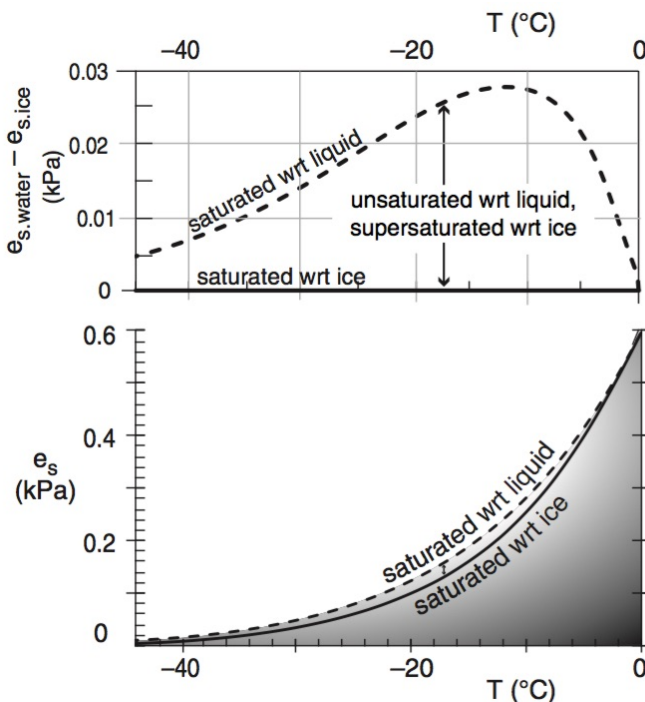
$L$  is the latent heat factor

$$R_v = 461 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$T_0 = 273.15 \text{ K}$$

The latent heat factor used in this equation depends on whether you are considering saturation over a liquid surface ( $L_v = 2.5 \times 10^6 \text{ J kg}^{-1}$ ) or an ice surface ( $L_d = 2.83 \times 10^6 \text{ J kg}^{-1}$ )

[Example](#): Consider unsaturated air. What happens to this air as it is cooled?



In the atmosphere liquid water can remain unfrozen at temperatures below 0 $^{\circ}\text{C}$  and as cold as -40 $^{\circ}\text{C}$ . Liquid water at a temperature below 0 $^{\circ}\text{C}$  is referred to as [supercooled water](#).

The saturation vapor pressure over an ice surface is slightly less than that over a liquid surface at the same temperature. [Why does this occur?](#)

The Clausius - Clayperon equation can also be used to calculate the vapor pressure ( $e$ ) using the dew point temperature ( $T_d$ )

$$e = e_0 \exp \left[ \frac{L}{R_v} \left( \frac{1}{T_0} - \frac{1}{T_d} \right) \right] \text{ (units: mb)}$$

The latent heat factor varies slightly with temperature, so the two forms of the Clausius-Clayperon equation above should use slightly different values of  $L$  depending on the temperature.

Instead, an approximation of the Clausius - Clayperon equation, known as **Teten's formula**, is often used to calculate vapor pressure and saturation vapor pressure.

$$e_s = e_0 \exp \left[ \frac{b(T - T_1)}{T - T_2} \right] \quad \text{and} \quad e = e_0 \exp \left[ \frac{b(T_d - T_1)}{T_d - T_2} \right]$$

$$e_0 = 6.113 \text{ mb}$$

$$b = 17.2694$$

$$T_1 = 273.15 \text{ K}$$

$$T_2 = 35.86 \text{ K}$$

## Moisture Variables

We can indicate the amount of moisture in the air in either absolute or relative terms.

**Absolute humidity:** A measure of the amount of water vapor contained in air. This can be expressed in several different ways.

Vapor pressure ( $e$ ) is one absolute measure of humidity.

**Mixing ratio ( $r$ ):** The mass of water vapor per unit mass of dry air

$$r = \frac{m_v}{m_d} \text{ (Units: kg kg}^{-1} \text{ or g kg}^{-1}\text{)}$$

$m_v$ : mass of water vapor

$m_d$ : mass of dry air

**Specific humidity ( $q$ ):** The mass of water vapor per unit mass of air

$$q = \frac{m_v}{m_d + m_v} = \frac{m_v}{m_{air}} \text{ (Units: kg kg}^{-1} \text{ or g kg}^{-1}\text{)}$$

$m_{air}$ : mass of air (dry air plus water vapor)

Which will have a larger value - mixing ratio or specific humidity?

The mixing ratio and specific humidity will not change as long as water vapor is not added to or removed from an air parcel.

We can also express humidity variables in terms of the saturated state of air:

**Saturation mixing ratio ( $r_s$ ):** The mass of water vapor in saturated air per unit mass of dry air

$$w_s = \frac{m_{v,s}}{m_d} \text{ (Units: kg kg}^{-1} \text{ or g kg}^{-1}\text{)}$$

$m_{v,s}$ : mass of water vapor in saturated air (Units: kg)

**Saturation specific humidity ( $q_s$ ):** The mass of water vapor in saturated air per unit mass of air

$$q_s = \frac{m_{v,s}}{m} \text{ (Units: kg kg}^{-1} \text{ or g kg}^{-1}\text{)}$$

**Relative humidity ( $RH$ ):** The ratio of the absolute amount of water vapor in the air to the absolute amount of water vapor required for the air to be saturated at that particular temperature .

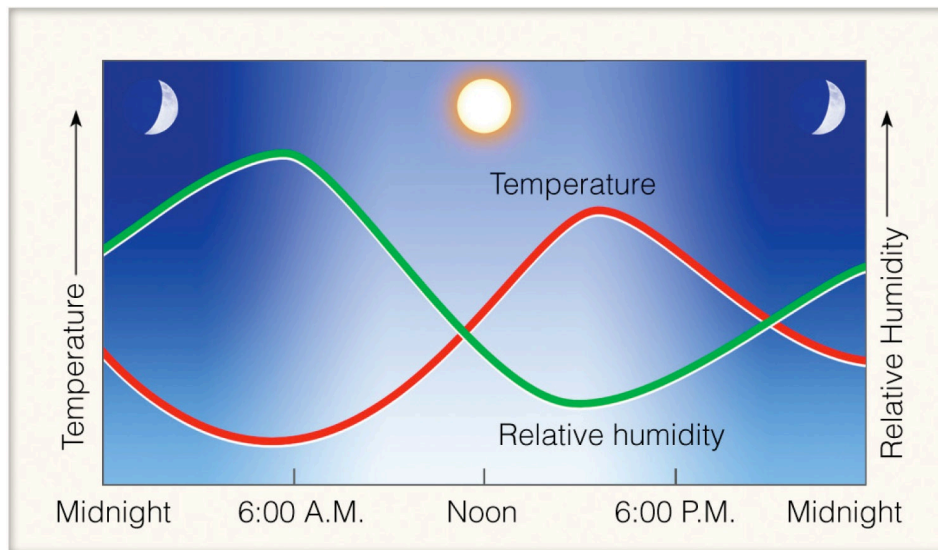
$$RH = \frac{e}{e_s} \times 100\% \approx \frac{r}{r_s} \times 100\% = \frac{q}{q_s} \times 100\% \text{ (Units: \%)}$$

The relative humidity ( $RH$ ) tells us how close the air is to being saturated but does not tell us directly how much water vapor is in the air (absolute humidity) or how much water vapor is required in order for the air to be saturated.

How will the relative humidity change if the amount of water vapor in the air increases?

How will the relative humidity change if the temperature of the air increases?

If the absolute amount of humidity does not change how will the relative humidity change over the course of a typical day?



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As unsaturated air cools its relative humidity will increase. The temperature at which the air has been cooled enough to cause the relative humidity to increase to 100%, and the air to become saturated, is called the **dew point temperature ( $T_d$ )**.

**Dew point temperature ( $T_d$ )**: The temperature to which air would have to be cooled (with no change in pressure or moisture content) for saturation to occur. (Units:  $^{\circ}\text{F}$ ,  $^{\circ}\text{C}$ , or  $\text{K}$ )

The dew point temperature is another absolute measure of humidity, like mixing ratio, specific humidity, and vapor pressure.

The table below shows the different ways we can express the actual amount of water vapor in the air (left column) or the amount of water vapor required for air to be saturated (right column):

Absolute measure of humidity	Absolute measure of saturation
$T_d$	$T$
$e$	$e_s$
$w$	$w_s$
$q$	$q_s$

Example: Which is more humid Las Vegas, NV in the summer or Bismarck, ND in the winter?

Las Vegas, NV

$T = 100^\circ\text{F}$

$T_d = 50^\circ\text{F}$

Bismarck, ND

$T = 0^\circ\text{F}$

$T_d = -10^\circ\text{F}$

## Converting between humidity variables

The dew point temperature is one of the most commonly reported measures of humidity and is often reported on weather maps.

If we know the dew point temperature we can calculate the vapor pressure using the Clausius - Clayperon or Teten's formulas given above.

We can convert between the various absolute measures of humidity using the following equations:

$$r = \frac{q}{1 - q} \quad q = \frac{r}{1 + r} \quad e = \frac{r}{\varepsilon + r} p$$

$$r = \frac{\varepsilon e}{p - e} \approx \frac{\varepsilon e}{p} \quad q = \varepsilon \frac{e}{p - (1 - \varepsilon)e}$$

where  $\varepsilon = 0.622$



Similar equations can be used to convert between the various saturation humidity variables:

$$\begin{aligned} r_s &= \frac{q_s}{1 - q_s} & q_s &= \frac{r_s}{1 + r_s} & e_s &= \frac{r_s}{\varepsilon + r_s} p \\ r_s &= \frac{\varepsilon e_s}{p - e_s} \approx \frac{\varepsilon e_s}{p} & q_s &= \varepsilon \frac{e_s}{p - (1 - \varepsilon)e_s} \end{aligned}$$

## Total Water

In addition to water vapor, water may exist as a liquid or solid (ice) in the atmosphere.

One way meteorologists quantify the amount of liquid water or ice in the atmosphere is with the [liquid water mixing ratio \( \$r\_L\$ \)](#) and the [ice water mixing ratio \( \$r\_i\$ \)](#).

$$r_L = \frac{m_L}{m_d} \quad r_i = \frac{m_i}{m_d} \quad (\text{Units: kg kg}^{-1} \text{ or g kg}^{-1})$$

$m_L$ : mass of liquid water

$m_i$ : mass of ice

The total amount of water in air is given [total water mixing ratio \( \$r\_T\$ \)](#) by:

$$r_T = r + r_L + r_i$$

For air that is unsaturated with no liquid water or ice this reduces to  $r_T = r$

## Moisture and Thermodynamic Diagrams

### Lagrangian Budgets

The rate of change of the total amount of water in an air parcel (its Lagrangian moisture budget) is expressed as:

$$\frac{\Delta r_T}{\Delta t} = S^{**}$$

where  $S^{**}$  is the net source or sink of total water in the air parcel

What is an example of a net source (sink) of total water in an air parcel?

For an air parcel that has no source or sink of total water ( $S^{**} = 0$ ) the total amount of water ( $r_T$ ) will be constant.

For this air parcel, over some period of time,  $r_{T,initial} = r_{T,final}$  or

$$(r + r_L + r_i)_{initial} = (r + r_L + r_i)_{final}$$

How will  $r$  change as an air parcel cools beyond its saturation point?

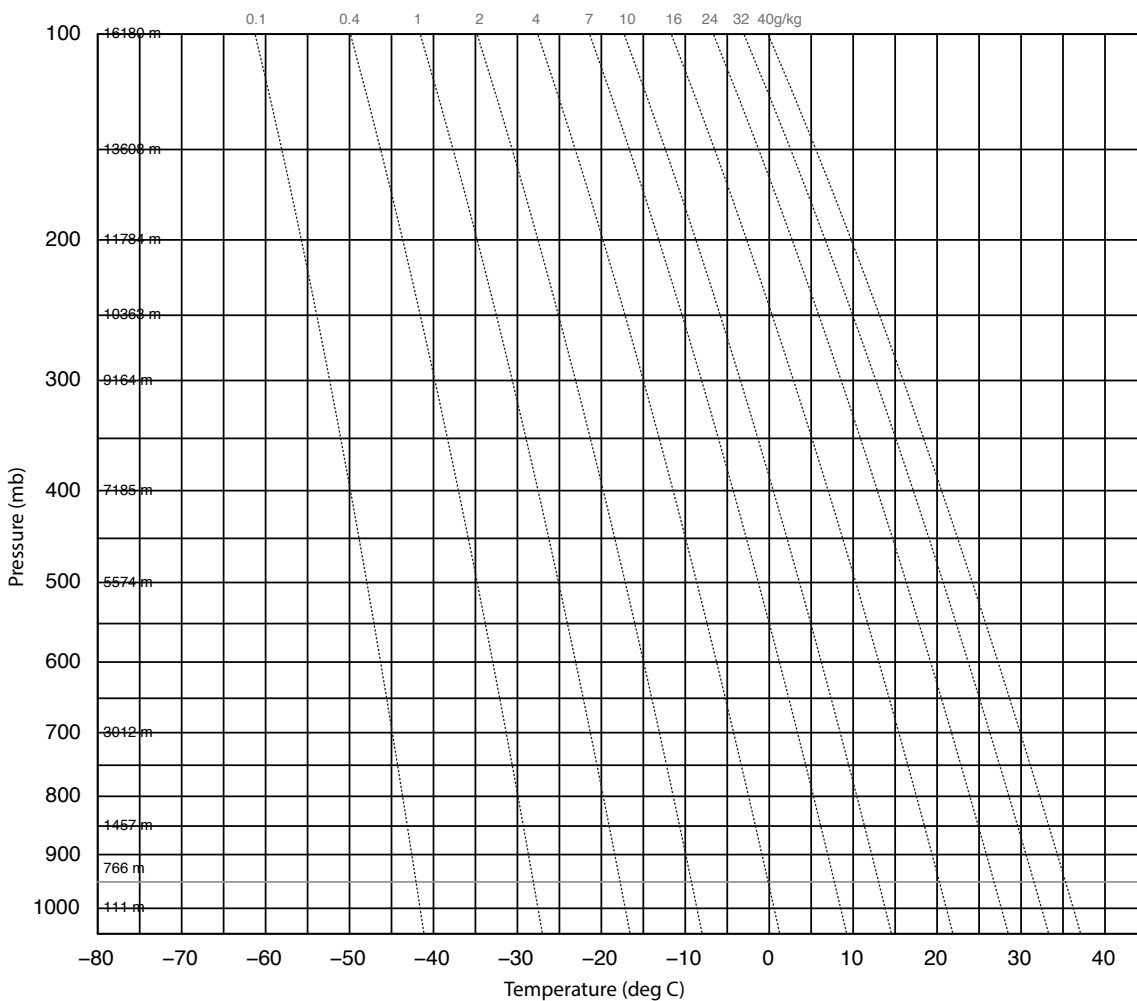
As  $r$  decreases as a saturated air parcel continues to cool  $r_L$  and / or  $r_i$  must increase to match the decrease in  $r$ .

Meteorologists often need to consider the vertical distribution of temperature and moisture in the atmosphere. To do this they will plot profiles of temperature and moisture on a thermodynamic diagram.

Lines of constant moisture (isohumes) can be plotted on a thermodynamic diagram. These isohumes are most often plotted as mixing ratios.

What will an isohume represent for a point on the diagram defined by an air parcel's temperature ( $T$ ) and pressure ( $p$ )?

What will an isohume represent for a point on the diagram defined by an air parcel's dew point temperature ( $T_d$ ) and pressure ( $p$ )?



On this diagram:

- lines of constant temperature are vertical (labeled on the bottom)
- lines of constant pressure are horizontal (labeled on the left side)
- lines of constant mixing ratio (or saturation mixing ratio) slope up and to the left and are plotted as dotted lines (labeled at the top of the diagram)

## Heat Conservation for Saturated Air

How will the relative humidity of an unsaturated air parcel change as it is lifted?

What happens to this air parcel as it lifted beyond the point where it first becomes saturated (its lifting condensation level)?

As a saturated air parcel rises water vapor condenses and releases latent heat. This latent heat offsets some of the adiabatic cooling that occurs as an air parcel rises, resulting in a slower rate of cooling for a saturated than an unsaturated air parcel.

**Saturated (or moist) adiabatic lapse rate ( $\Gamma_s$ ):** The rate at which a saturated air parcel will cool as it rises.

As was done for the dry adiabatic lapse rate, the first law of thermodynamics can be used to determine the moist adiabatic lapse rate.

$$\Delta q = C_p \Delta T - \alpha \Delta p$$

For a moist adiabatic process  $\Delta q$  is equal to the amount of latent heat released as water vapor condenses or the amount of latent heat consumed as liquid water evaporates.

The amount of latent heat released or consumed can be calculated by multiplying the latent heat of condensation / evaporation ( $2.5 \times 10^6 \text{ J kg}^{-1}$ ) by the mass of water that is changing phase.

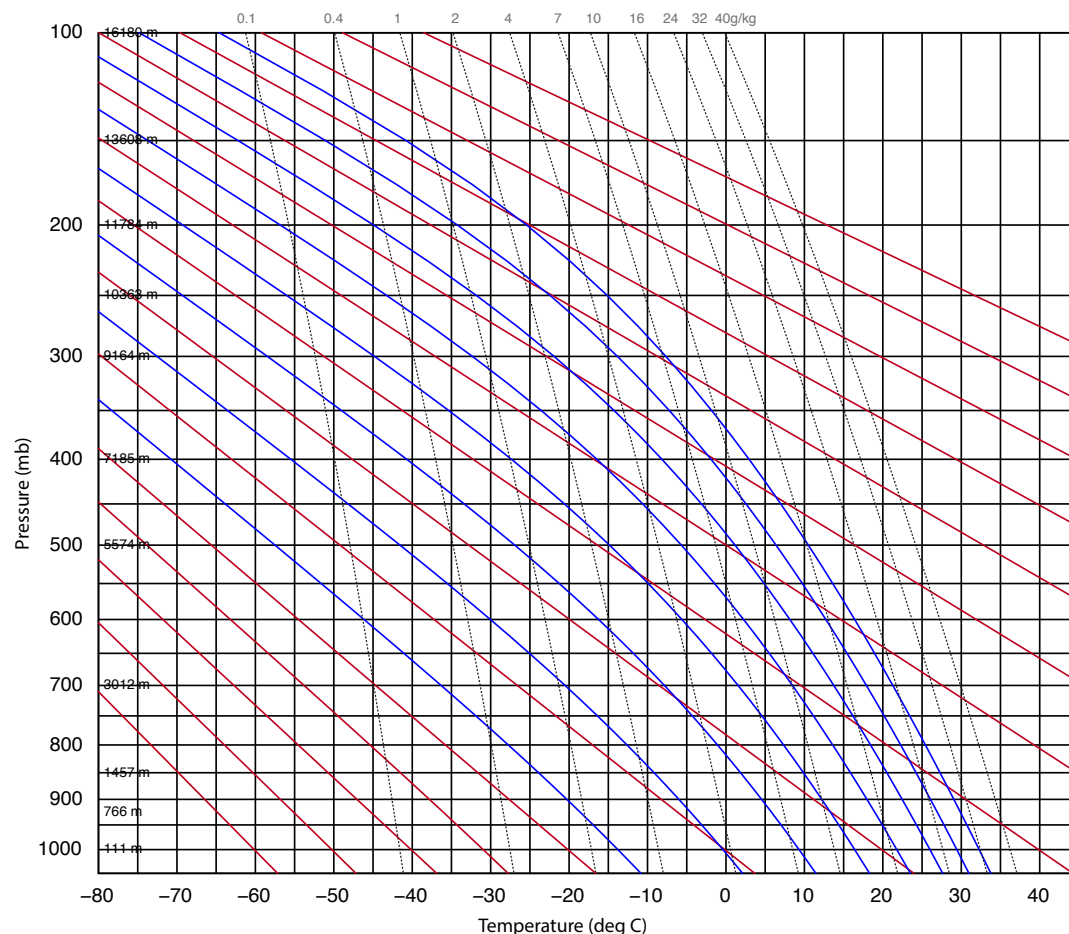
If we assume that our air parcel maintains a 100% relative humidity during this process the mass of water changing phase per 1 kg of air is equal to the change in the saturation mixing ratio ( $\Delta r_s$ ). This allows us to calculate the moist adiabatic lapse rate as:

$$\Gamma_s = \frac{\Delta T}{\Delta z} = \frac{|g|}{C_p} \frac{1 + \frac{r_s L_v}{R_d T}}{1 + \frac{L_v^2 r_s \epsilon}{C_p R_d T^2}} \quad \text{or} \quad \frac{\Delta T}{\Delta p} = \frac{(R_d/C_p)T + (L_v/C_p)r_s}{P \left( 1 + \frac{L_v^2 r_s \epsilon}{C_p R_d T^2} \right)}$$

The moist adiabatic lapse rate varies with the amount of moisture in the air parcel and typically has values of 4 to 7 K km<sup>-1</sup>.

How does the value of the moist adiabatic lapse rate compare to the magnitude of the dry adiabatic lapse rate?

Using these equations we can plot moist adiabats (also called saturated adiabats) on a thermodynamic diagram.



On this diagram:

- lines of constant temperature are vertical (labeled on the bottom)
- lines of constant pressure are horizontal (labeled on the left side)
- lines of constant mixing ratio or saturation mixing ratio are dotted and slope up and to the left (labeled at the top)
- dry adiabats are red lines that slope up and to the left
- moist adiabats are curved blue lines that slope up and to the left

The value of both the dry and moist adiabats can be determined by finding the temperature at which these lines cross the 1000 mb pressure line.

We can use this diagram to graphically depict how an air parcel's temperature will change as it moves either dry or moist adiabatically through the atmosphere.

### Wet bulb and equivalent potential temperature

How does the potential temperature of an unsaturated air parcel change for an adiabatic process?

What happens to the potential temperature of this air parcel once it is lifted above its lifting condensation level?

For saturated adiabatic processes we can define an **equivalent potential temperature** ( $\theta_e$ ), which will not change for either unsaturated or saturated adiabatic processes.

$$\theta_e = \theta + \left( \frac{L_v \theta}{C_p T} \right) r$$

As an air parcel rises above its LCL the potential temperature ( $\theta$ ) will increase and the mixing ratio ( $r$ ) will decrease and these changes in the two terms on the RHS of the equation above will balance.

What does equivalent potential temperature represent physically?

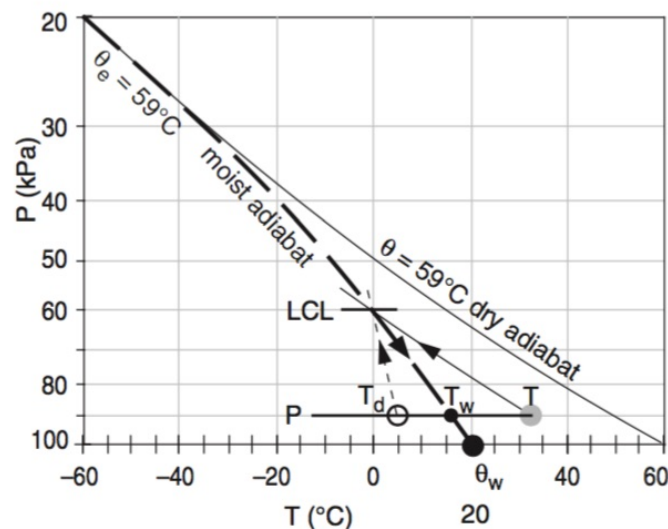
How does this differ from potential temperature?

The moist adiabats plotted on thermodynamic diagrams are labeled with values of **wet bulb potential temperature** ( $\theta_w$ ).

The wet bulb potential temperature for an air parcel is given by the value of the moist adiabat that passes through the lifting condensation level of the air parcel.

At a given pressure the wet bulb temperature ( $T_w$ ) of an air parcel is given by the temperature of the moist adiabat that passes through the LCL at that pressure.

The relationship between pressure ( $p$ ), temperature ( $T$ ), dew point temperature ( $T_d$ ), wet bulb temperature ( $T_w$ ), potential temperature ( $\theta$ ), wet bulb potential temperature ( $\theta_w$ ), and equivalent potential temperature ( $\theta_e$ ) is shown below.



## Water Budget at a Fixed Location

What processes can alter the amount of water at a fixed location in the atmosphere?

$$\frac{\Delta r_T}{\Delta t} = - \left[ U \frac{\Delta r_T}{\Delta x} + V \frac{\Delta r_T}{\Delta y} \right] + \left( \frac{\rho_L}{\rho_d} \right) \frac{\Delta Pr}{\Delta z} - \frac{1}{\rho} \frac{\Delta F_{z,turb}(r_T)}{\Delta z} \quad (\text{Units: kg}_{\text{water}} \text{ kg}_{\text{air}}^{-1} \text{ s}^{-1})$$

$r_T$  is the total water mixing ratio

$Pr$  is the precipitation rate (see below for units)

$\rho_L$  and  $\rho_d$  are the density of liquid water ( $1000 \text{ kg m}^{-3}$ ) and air ( $\sim 1 \text{ kg m}^{-3}$ )

$F_{z,turb}(r_T)$  is the turbulent flux (transport) of total water ( $\text{kg}_{\text{water}} \text{ m}^{-2} \text{ s}^{-1}$ )

(Note: Unlike the textbook the symbol  $F$  represents the flux not the kinematic flux)

What physical process does each term in this equation represent?

Similar to temperature advection, moisture advection occurs when there is wind blowing across a gradient in total water mixing ratio.

$Pr$  is often given with units of depth (mm, cm, etc.) per hour or day. For use in the equation above this depth per time should be converted to  $\text{m s}^{-1}$ .

**Liquid water equivalent:** The amount of liquid water depth that results from melting a given amount of snow or ice.

If snowfall is being measured, the amount of snow needs to be converted to a **liquid water equivalent** before being used in this equation.

$\Delta Pr / \Delta z$  indicates the difference in the amount of precipitation falling through the top and bottom edges of the fixed volume of air being considered.

Turbulent transport of water occurs when small-scale turbulent motions mix air with different water contents.

The turbulent moisture flux divergence ( $\Delta F_{z,turb}(r_T) / \Delta z$ ) indicates the net transport of water into or out of the volume of air due to turbulent motions.

### Moisture flux at the Earth's Surface

In chapter 3 we consider the heat budget at the surface of the Earth, which include the latent heat flux ( $F_E$ ).

The latent heat flux is directly related to the moisture flux by  $F_E = \frac{F_{water}}{L_v}$

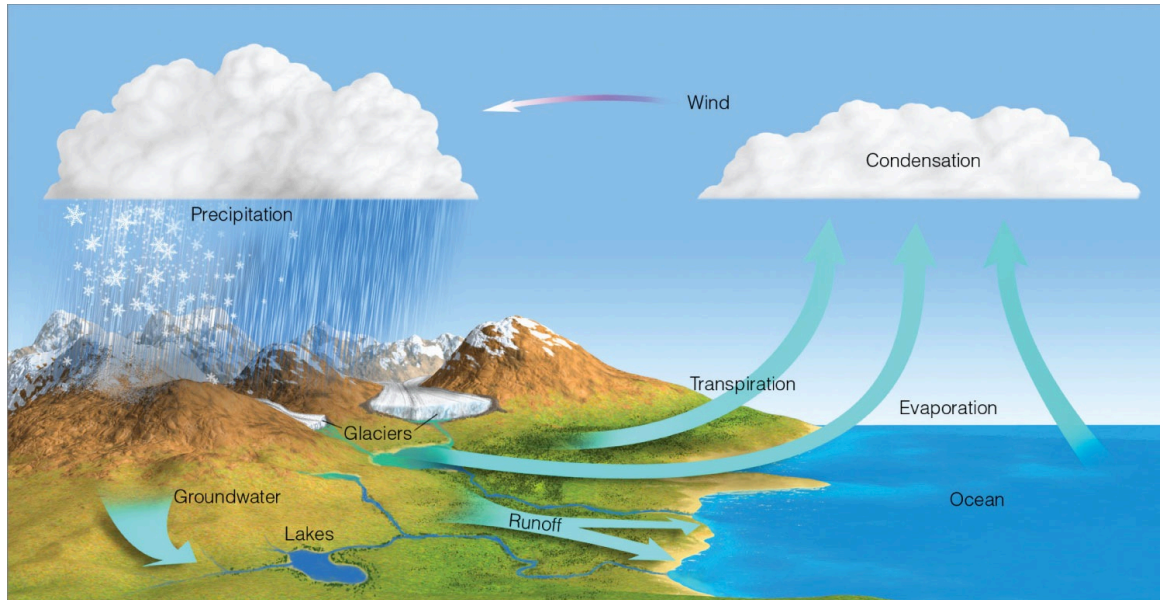
The surface turbulent water flux can be estimated in a manner similar to the surface turbulent heat flux in chapter 3.

$$F_{water} = C_H M (r_{sf} - r_{air})$$

where  $M$  is the wind speed at 10 m above the ground and  $C_H$  is a bulk transfer coefficient with values between  $2 \times 10^{-3}$  and  $2 \times 10^{-2}$ .



**Hydrologic cycle:** The cycle of water moving through the climate system (ocean, atmosphere, land) and its transformation between liquid, vapor, and solid phases.



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