Physical Meteorology, MTR3440
Spring 2008
Homework #4
Due Wed., April 9

1. Consider an activated cloud droplet with a radius of 1.5 μm in a cloud that is at 700 mb and 10°C, with a saturation ratio of 1.005.
    a. How long (in days) would it take this droplet to grow to be a precipitation-sized droplet 1 mm in radius (Hint: Use Figure 7.1, R&Y)? Given this amount of time, do you expect that condensational growth can account for the formation of precipitation in clouds?

Here we need to solve the growth equation for time,

$$t = \frac{r^2 - r_0^2}{2\xi} = \frac{r^2 - r_0^2}{2\xi_1 S}$$

Reading from Figure 7.1, we get $\log_{10}\xi_1=2$ at 70 kPa and T=10 °C, therefore $\xi_1 = 100\ \mu m^2\ s^{-1}$. Plugging the numbers in, we get

$$t = \frac{1000\ \mu m^2 - 150\ \mu m^2}{2\left(100\ \mu m^2 s^{-1}\right)} \frac{1\ day}{86400\ s} = 11.6\ days$$

Given this amount of time, one wouldn't expect that condensational growth can account for the formation of precipitation in clouds

    b. How long would it take this droplet to grow to a radius of 20 μm? For particles around 20 μm, the collision-coalescence process starts to be an efficient mechanism for particle growth. Does the time for the drop to grow to 20 μm make you think that the precipitation will be able to form in a reasonable amount of time?

Solving for the time as in part a,

$$t = \frac{1000\ \mu m^2 - 150\ \mu m^2}{2\left(100\ \mu m^2 s^{-1}\right)} \frac{1\ minute}{60\ s} = 6.6\ minutes$$

This indicates that drops can grow to a size of 20 μm in a reasonable amount of time. Given that collision-coalescence takes over once drops reach this size, it is reasonable to think that the combination of these two processes can account for the formation of precipitation.

2. A drop with an initial radius of 100 μm falls through a cloud containing 100 droplets per cubic centimeter that it collects in a continuous manner with a collection efficiency of 0.8. If all the cloud droplets have a radius of 10 μm, how long will it take for the drop to reach a radius of 1 mm? You may assume that for the drops of the size considered in this problem the terminal fall speed v (in m s^{-1}) of a drop of radius r (in m) is given by $v = 8 \times 10^3\ r$. Assume that the cloud droplets are stationary and that the updraft speed in the cloud is negligible.

The equation for droplet growth by collision-coalescence is

$$\frac{dR}{dt} = \frac{\bar{E}M}{4\rho_l} u(R),$$

where $\bar{E}$ is the average collection efficiency, M is the liquid water content, and u(R)=v, the terminal velocity. Plugging in the equation for terminal velocity above, and integrating, we get

$$\int_{r_1}^{r_2} \frac{dR}{u(R)} = \int_0^t \frac{\bar{E}M}{4\rho_l} dt$$

$$\int_{r_1}^{r_2} \frac{dR}{ar} = \int_0^t \frac{\bar{E}M}{4\rho_l} dt$$

$$\left(\ln r_2 - \ln r_1\right) = \frac{\bar{E}Ma}{4\rho_l} t$$

$$t = \left(\ln r_2 - \ln r_1\right) \frac{4\rho_l}{\bar{E}Ma}$$

$$M = \frac{4}{3}\pi r^3 \rho_l N$$

$$\therefore t = \left(\ln r_2 - \ln r_1\right) \frac{3}{\bar{E}\pi r^3 Na}$$

$$t = \left(\ln(0.1\text{cm}) - \ln(0.01\text{ cm})\right) \frac{3}{0.8\pi(0.001cm)^3(100cm^{-3})(8x10^3 s^{-1})} = 3436\,\text{sec}\,\frac{1\,\text{min}}{60\,\text{sec}} = 57\,\text{min}$$

3. If the raindrop in the question above exits the cloud base, which is located 5 km above the ground, what will be its radius at the ground and how long will it take to reach the ground if the relative humidity between cloud base and ground is constant 60%? To calculate this, you will use the growth equation, assuming $\xi_1 = 700\ \mu m^2\ s^{-1}$. Use the equation in problem 2 for the terminal velocity.

To solve this problem, you need to start with the definition of velocity, which is

$$v = \frac{dz}{dt}$$

$$dz = vdt$$

Here we will assume that as the drop falls, it's radius will change as it evaporates, but it's velocity is always given by it's terminal velocity, or v = ar, where a = 8 x 10^3 s^{-1}. Thus,

$$dz = ardt$$

We will also assume that the radius is given by the growth equation describes how the radius changes with time, $r(t) = \sqrt{r_0^2 + 2\xi t}$. Plugging this in to our equation for dz, we get

$$dz = a\sqrt{r_0^2 + 2\xi t}dt$$

Integrating this with appropriate limits

$$\int_0^{z_{ground}} dz = \int_0^{t_{ground}} a\sqrt{r_0^2 + 2\xi t}dt$$

$$z_{ground} = \frac{a}{3\xi}\left[\left(r_0^2 + 2\xi t_{ground}\right)^{3/2} - r_0^3\right]$$

$$\Rightarrow t_{ground} = \frac{1}{2\xi}\left[\left(\frac{3\xi z_{ground}}{a} + r_0^3\right)^{2/3} - r_0^2\right]$$

In this problem, S=0.6 (60% RH), so ξ=(0.6-1)(700 $\mu m^2 s^{-1}$)= -280 $\mu m^2 s^{-1}$. Plugging this in to the equation, we get

$$\Rightarrow t_{ground} = \frac{1}{2(-280\mu m^2 s^{-1})}\left[\left(\frac{3(-280\mu m^2 s^{-1})(5x10^9\,\mu m)}{8x10^3\,s^{-1}} + (1000\mu m)^3\right)^{2/3} - (1000\mu m)^2\right] = 700\,\text{sec}$$

Now, using this time in the growth equation, we get

$$r = \sqrt{r_0^2 + 2\xi t} = \sqrt{(1000\mu m)^2 + 2(-280\mu m^2 s^{-1})(700 s)} = 780\mu m$$

4. In this problem, you will estimate the mass of precipitation between the ground and cloud base (per unit surface area on the ground), assuming a cloud with a base 5 km above ground. To do this, consider a 1 m^2 patch of ground. Imagine you are watching this patch during a rainstorm. Based on your experience, how many drops per second will hit this patch? This number gives you the "flux" of raindrops, in # m^{-2} sec^{-1}. You will then divide this number by the terminal velocity of the raindrops (look it up in a table, using a reasonable size for raindrops) to obtain the concentration of raindrops per unit volume (# m^{-3}). Using an appropriate size for raindrops, convert this concentration to a LWC (g m^{-3}). Finally, multiply this by the volume of air (between the ground and cloud base) per unit surface area on the ground (units of this are m^3 m^{-2}). This final number is the mass of precipitation between you and the cloud, per unit surface area on the ground (g m^{-2}). How does this mass compare to the mass of air above you? To do this you will need to calculate the mass of atmosphere per unit surface area (g m^{-2}).

Here we'll assume 50 drops hit one square meter of ground per unit second, and that they have 1 mm diameter. Drops with 1 mm diameter have a terminal velocity of 4 m s^{-1}, from Table 8.1. Thus the volume swept out by drops is (4 m s^{-1})*(1 m^2) = 4 m^3 s^{-1}, and the density of drops is (50 drops s^{-1})/(4 m^3 s^{-1}) = 12.5 drops m^{-3}. Each drop has a mass of ($4/3*\pi*r^3*\rho_L$=5.23x10^{-4} g), so the LWC is 0.0065 g m^{-3}. Multiplying by 5000 m^3 m^{-2}, we get 32.7 g m^{-2}. The mass of the air above is given by the pressure divided by g. m/A = P / g = (1.013 x 10^5 N m^{-2}) / (9.8 m s^{-2}) = 10336 kg m^{-2}, which is much higher than the amount of precipitation…