Physical Meteorology, MTR3440
Spring 2008
Homework #3
**Due Wed., March 19**

1. Rogers and Yau, Problem 5.1

Cloudy air consists of dry air, water vapor, and cloud droplets. In a particular cloud volume, there are 200 droplets per cubic centimeter, all of the same size, with a radius of 10 μm. The temperature is 10°C and the pressure is 80 kPa. Determine the following properties of the cloud:

a. The mass of cloud water per unit volume.

$$\text{LWC}=\frac{\rho_{\text{liquid}} N V_{\text{drop}}}{V_{\text{air}}}=\frac{(1\text{ g cm}^{-3})(200)(\frac{4}{3}\pi(10\text{ x }10^{-4}\text{ cm})^3)}{1\text{ cm}^3}\frac{10^6\text{cm}^3}{\text{m}^3}=0.84\text{ g m}^{-3}$$

b. The mass of water vapor per unit volume

To solve this you need to use some form of the Clausius-Clapeyron equation to calculate the partial pressure of water vapor (assuming 100% RH in the cloud) at the given temperature of 283 K. The partial pressure is 12 mb. Using the ideal gas law to solve for density, we get

$$\rho=\frac{p}{RT}=\frac{1200\text{ Pa}}{(462\text{ J kg}^{-1}\text{ K}^{-1})(283\text{ K})}\frac{10^3\text{g}}{\text{kg}}=9.4\text{ g m}^{-3}$$

c. The mass of dry air per unit volume

This one is similar to above, except we're using the total pressure (which is essentially the same as the pressure of dry air)

$$\rho=\frac{p}{RT}=\frac{80000\text{ Pa}}{(287\text{ J kg}^{-1}\text{ K}^{-1})(283\text{ K})}=0.985\text{ kg m}^{-3}$$

d. The mean distance between the droplets

There are several ways to solve this, any of which I will accept. The way the book does this is to assume that each drop takes up an equal spherical volume of air, and that the distance between the drops is twice the radius of the volume for each drop. Assuming each droplet occupies an equal volume of air, the volume is equal to (1 $\text{cm}^3$ / 200 = 0.005 $\text{cm}^3$). Then solving for twice the radius of this spherical volume

$$\text{distance}=2r=2\left(\frac{3}{4\pi}V\right)^{1/3}=2\left(\frac{3}{4\pi}(0.005\text{ cm}^3)\right)^{1/3}=0.21\text{ cm}$$

2. The 2D-S particle probe is an instrument that can be mounted on research aircraft to take extremely high speed images of cloud particles. From these images, the particle size distribution is calculated. An average particle size distribution from the 2D-S probe taken within so-called "subvisual" cirrus can be found at http://atoc.colorado.edu/~seand/class/MTR3440/hw2prob3.xls. The data are stored in terms of # of particles $L^{-1}$ $bin^{-1}$. The left column is the bin edge *diameter*. So for example, the first bin goes from 10-20 μm, and the number concentration of particles within that bin is 2.02 $L^{-1}$.

   The spreadsheet with all data for this problem can be downloaded from http://atoc.colorado.edu/~seand/class/MTR3440/hw2prob3soln.xls

   a. Calculate the total number concentration (in units # $cm^{-3}$) from the size distribution.

   2727 $cm^{-3}$

   b. Assuming these particles are spheres, calculate the extinction coefficient (in units of $km^{-1}$). You can assume the extinction efficiency is 2. The equation for extinction is $\beta = \int Q_{ext} \pi r^2 N(r) dr$, but you will need the discrete form $\beta = Q_{ext} \pi \sum r^2 n(r)$

   0.00109 $km^{-1}$

   c. Assuming these particles are spheres, calculate the ice water content (in units g $cm^{-3}$). Once again, you will need the discrete form of the equation for IWC, $IWC = \rho_{ice} \int \frac{4}{3} \pi r^3 N(r) dr$. I'll leave this one for you to figure out on your own.

   8.15 x $10^{-12}$ g $cm^{-3}$

   d. Assuming the cloud is 0.5 $km^{-1}$ thick, calculate the optical thickness of the cloud from your answer in part b. Given that the human eye can detect things with an optical thickness of about 0.03, does this cloud qualify as being "subvisual"?

   $\tau = \beta z = (0.00109\ km^{-1})(0.5\ km) = 0.000545$

   e. Plot the dN/d(logD) size distribution on a log-log scale. What are the units of dN/d(logD)?

   # $L^{-1}$ – d(logD) is unitless

   f. Are there more particles of size 310 μm or 360 μm? Explain your answer.

   There are 1 x $10^{-9}$ particles per liter in the bin from 300-320 μm, whereas there are 2 x $10^{-9}$ particles per liter in the bin from 320 – 400 μm. Just by looking at this, one might say that there are more particles at 360 μm than 310 μm. However, because the bins are of different size, we can't make this simple comparison. We need to consider the value of

the size distribution in terms of # $L^{-1}$ $\mu m^{-1}$. To do this, we divide the concentrations above by the bin widths. The values for the 300-320 μm and 320 – 400 μm bins are 5 x $10^{-11}$ $L^{-1}$ $\mu m^{-1}$ and 2.5 x $10^{-11}$ $L^{-1}$ $\mu m^{-1}$, respectively. From this, we see that there are more particles at 310 μm than 360 μm.

3. Using the approximate equation for the saturation ratio 6.6 in your book, show that the critical radius, $r^*$, and critical saturation ratio, $S^*$, are given by

$$r^* = \sqrt{3b/a}$$

$$S^* = 1 + \sqrt{4a^3/27b}$$

The critical radius occurs when dS/dr=0. So one needs to take the derivative of S(r), and solve for r to get $r^*$. Then one plugs $r^*$ into the equation for S to get $S^*$.

4. Over the ocean, sea salt aerosol (NaCl) is generated when sea spray from breaking waves evaporates. These aerosol particles may act as cloud condensation nuclei when they are lofted to levels at or above saturation. Assume a salt particle of mass $10^{-16}$ g is at a temperature of 10°C.

a. What radius must the droplet attain to become "activated"?

We need to solve for $r^*$ here, using the equation from problem 3.

$$r^* = \sqrt{3b/a}$$

From page 88 in Rogers and Yau, we see the equations for a and b

$$a = \frac{3.3 \text{ x } 10^{-5} \text{ cm K}}{T} = \frac{3.3 \text{ x } 10^{-5} \text{ cm K}}{283 \text{ K}} = 1.17 \text{ x } 10^{-7} \text{ cm}$$

$$b = \frac{4.3iM}{m_s} = \frac{4.3(2)(10^{-16}\text{g})}{58.5 \text{ g mol}^{-1}} = 1.47 \text{ x } 10^{-17} \text{ cm}^3$$

$$r^* = \sqrt{3b/a} = \sqrt{3(1.47 \text{ x } 10^{-17} \text{ cm}^3)/(1.17 \text{ x } 10^{-7}\text{cm})}\,\frac{10^4\,\mu m}{cm} = 0.20\mu m$$

b. If the supersaturation is 0.5%, will this droplet become activated?

We need to compare the supersaturation of 0.5% (S=1.005) to the critical supersaturation, $S^*$. If $S > S^*$, then this droplet will become activated.

$$S^* = 1 + \sqrt{4a^3/27b} = 1 + \sqrt{4(1.17x10^{-7}cm)^3/(27(1.47x10^{-17}cm^3))} = 1.0040$$

Since $S < S^*$, the drop does not become activated.

5. Calculate the critical radius of a droplet of <u>pure water</u> at 0°C for saturation ratios of 1.02, 2.5, and 7.5

For pure water, the critical radius is given by

$$r_c = \frac{2\sigma}{R_v \rho_l T \ln S}$$

Solving for $r_c$, we get

$$r_c = \frac{2\sigma}{R_v \rho_l T \ln S} = \frac{2(7.5\text{x}10^{-2}\ \text{N / m})}{(462\ \text{J kg}^{-1}\ \text{K}^{-1})(1\ \text{g cm}^{-3})(10^{-3}\ \text{kg g}^{-1})(10^{6}\ \text{cm}^{3}\ \text{m}^{-3})(273\ \text{K})(\ln 1.02)} \frac{10^6 \mu\text{m}}{\text{m}} = 0.06\mu\text{m}$$

Similarly, we get $r_c$=0.0013 μm for S=2.5 and $r_c$=0.00059 μm for S=7.5.