1. According to the 2007 Fourth Assessment Report of the Intergovernmental Panel on Climate Change, aviation-induced contrails are responsible for a radiative forcing of 0.01 W m\(^{-2}\) (compared to ~1.6 W m\(^{-2}\) of human-caused forcing) since the beginning of the industrial era. Explain how these high-altitude clouds (compared to the clear sky) can alter the top-of-atmosphere energy balance during the day, and at night.

During the day, contrail cirrus can reduce the net TOA SW radiation going into Earth by reflecting sunlight back to space. During both day and night, they decrease the outgoing longwave radiation (OLR). These two effects act to offset one another in terms of their net effect on the TOA balance.

2. Based on your answer to 1, do you think contrail clouds will contribute to a larger positive forcing during the day, or at night? Why?

A positive forcing means that more energy is entering the TOA when the cirrus are present, compared to the clear sky. During the day, the SW “albedo” effect and LW “greenhouse” effect act to offset one another. At night, only the LW effect is at work, so contrail clouds are more likely to have a positive forcing at night than during the day.

3. Averaged over a year, regions of Earth poleward of about 40° latitude emit more radiation than they absorb (at the top of atmosphere). Conversely, regions equatorward of 40° absorb more radiation than they emit. One might (erroneously) think that because of this, the equatorial regions should heat up indefinitely, while the polar regions cool indefinitely. However, we know that the equator-to-pole temperature gradient is relatively stable over longer timescales. What are the two mechanisms responsible for preventing the equator-to-pole temperature gradient from “running away”?

The atmosphere, and the oceans.
4. If you average the net radiative heating in the troposphere over the globe, cooling from emission is greater than heating from absorption. What mechanisms of heat transfer balance the net radiative cooling in the troposphere?

Several mechanisms balance the net radiative cooling of the troposphere. Latent heating from water vapor that is evaporated at the surface releases heat upon condensing in clouds. Also, atmospheric motions, particularly convection, contribute to heating of the troposphere by moving heat from the surface upwards.

5. Without using the equation for solar zenith angle, what is the solar zenith angle at solar noon at the South Pole on Dec 21? Draw a diagram of the Earth and Sun to illustrate your answer, and explain your reasoning.

December 21 is the summer solstice for the southern hemisphere, and the declination angle is −23.5°. This means that at a latitude of 23.5° S, the sun is directly overhead (i.e., SZA = 0°) on Dec 21. If we go then to 90° S at the S. Pole, the sun is at a zenith angle of 

\[ Z = 90° - \delta = 90° - 23.5° = 66.5°. \]

6. Describe qualitatively what is meant by the term “weighting function”, and how it is related to the vertical coordinate optical depth.

Weighting functions are defined as the vertical derivative of transmittance between the location of the atmosphere and “space”, or \( \tau = 0 \). Qualitatively, weighting functions tell us at what altitude radiation at a specific wavelength (or averaged over a “channel” of wavelengths) is emitted from, or where it is absorbed. From the standpoint of radiation received by a satellite above Earth’s atmosphere, we know that most radiation comes from the level of \( \tau = 1 \). Because of this, weighting functions are typically centered around \( \tau = 1 \).
7. The Cloud Profiling Radar (CPR) aboard NASA’s CLOUDSAT satellite operates at 94 GHz. What “regime” of interaction do you expect this radiation to have with cloud droplets that are 10 μm in diameter? What about precipitation droplets that are 2 mm in diameter? Which of these types of particles do you expect to scatter more radiation back to the satellite sensor, and why?

In this problem we need to calculate the size parameters, $x = \frac{2\pi r}{\lambda}$, for the two types of particles. But first, we need to calculate the wavelength

$$\lambda = \frac{c}{f} = \frac{2.99 \times 10^8 \text{ m s}^{-1}}{94 \times 10^9 \text{ s}^{-1}} = 3.2 \times 10^{-3} \text{ m} = 3.2 \text{ mm}$$

For the 10 μm particle, the size parameter is

$$x = \frac{2\pi r}{\lambda} = \frac{2\pi (10 \times 10^{-6} \text{ m})}{3.2 \times 10^{-3} \text{ m}} = 0.02$$

So for the 10 μm particle, the radiative interactions are in the Rayleigh scattering regime. For the 2 mm droplet, the size parameter is

$$x = \frac{2\pi r}{\lambda} = \frac{2\pi (2 \times 10^{-3} \text{ m})}{3.2 \times 10^{-3} \text{ m}} = 3.9$$

so it is in the Mie scattering regime. We expect the particles in the Mie regime to scatter more radiation. Recall the plot of scattering efficiency versus size parameter, in which the value is very low in the Rayleigh regime, is very oscillatory in the Mie regime. Another way of thinking about this is to think about what the subject matter of the question. We are talking about a radar, which you might recall is sensitive to precipitation-sized droplets, and not as much to normal cloud particles.

8. The color that we perceive a star to be corresponds roughly to it’s wavelength of peak emission. Given this, do you expect a blue star or a red star to be hotter? Why?

We expect blue stars to be hotter. From Wein’s law, as the temperature goes UP, the wavelength of peak emission goes DOWN. Thus, stars with lower (bluer) wavelengths of peak emission are hotter than stars with higher (redder) wavelengths.

9. Circle the four most important determining factors in determining the value of the daily solar insolation at a given location on Earth:

- Latitude
- Longitude
- Atmospheric Brightness
- Temperature
- Solar Azimuth Angle
- Solar Declination Angle
- Earth Radius
- Earth-Sun Distance
- Sunrise Time
- Planetary Albedo
- Solar Constant

3
10. Match the following satellite channels from the GOES imager to the wavelength bands indicated:

- **Thermal IR Channel**: 0.5 – 0.7 µm
- **Shortwave IR Channel**: 11.5 – 12.5 µm
- **Visible Channel**: 3.8 – 4.0 µm
- **Water Vapor Channel**: 6.5 – 7.0 µm

11. Solar radiation ($S_0=1360 \text{ Wm}^{-2}$) enters the top of the atmosphere with a zenith angle of $20^\circ$. Suppose the attenuation of solar radiation by air is independent of wavelength with an extinction coefficient $k = 2.5 \times 10^{-4} \text{ cm}^2\text{g}^{-1}$. What is the optical depth of this atmosphere? Determine the solar irradiance in Denver (~840 mb).

Recall the appropriate equation for optical depth,

$$\tau = \int_{z_i}^{z_f} kr \rho dz$$

Here we need to recognize that the vertical coordinate is pressure, so we substitute in the hydrostatic equation

$$dp = -\rho g dz$$

$$\tau = \int_{p_1}^{0} \frac{kr}{g} dp$$

Recognizing that the mixing ratio of air is 1, and that $k$ is independent of altitude, we get

$$\tau = \int_{p_1}^{0} \frac{kr}{g} dp = \frac{k}{g} (p_1 - p_2) = \frac{2.5 \times 10^{-4} \text{ cm}^2\text{g}^{-1}}{9.8 \text{ms}^{-2}} \times \frac{1 \text{m}^2}{10^4 \text{cm}^2} \times \frac{10^3 \text{g}}{\text{kg}} \left( 84000 \text{Nm}^{-2} - 0 \text{Nm}^{-2} \right) = 0.21$$

Now, we need to solve for the irradiance in Denver

$$F = F_0 e^{-\mu}$$

In this case $F_0$ is $(1360 \text{ Wm}^{-2} \times \cos 20) = 1278 \text{ Wm}^{-2}$

$$F = F_0 e^{-\mu} = 1278 \text{ Wm}^{-2} e^{-0.21} = 1022 \text{ Wm}^{-2}$$

12. Betelgeuse, a red giant, is the brightest star in the Orion constellation. Its monochromatic intensity peaks at 0.8 µm. Determine the rate of radiative energy emitted from a square meter of its surface.

Recall that the total rate of energy emitted by a blackbody surface is given by Boltzmann’s law, $F = \sigma T^4$

Here we need to know $T$, but all we have is the wavelength at which the peak emission occurs. We need to use Wien’s law to get the temperature.
\[ T = \frac{2897 \mu m K}{0.8 \mu m} = \frac{2897 \mu m K}{0.8 \mu m} = 3620 K \]

Now that we have the temperature, we can easily calculate the emitted energy

\[ F = \sigma T^4 = 5.67 \times 10^{-8} W m^{-2} K^{-4} (3620 K)^4 = 9.7 \times 10^6 W m^{-2} \]

13. The heating rate within a cloud can be inferred by measuring the downwelling and upwelling irradiances both below and above the cloud. One way this can be done is by flying an aircraft with special instruments below and above the cloud.

Given the data below, calculate the radiative heating rate (in °C hr⁻¹) within the cloud layer. Assume the density of air is 0.5 kg m⁻³ at this altitude.

\[ F_{SW ↓} = 1200 W m^{-2} \quad F_{SW ↑} = 600 W m^{-2} \quad F_{LW ↑} = 200 W m^{-2} \quad z = 12 \text{ km} \]

\[ F_{SW ↓} = 300 W m^{-2} \quad F_{SW ↑} = 200 W m^{-2} \quad F_{LW ↓} = 250 W m^{-2} \quad F_{LW ↑} = 320 W m^{-2} \quad z = 11 \text{ km} \]

The equation for heating rate is \( \frac{dT}{dt} = \frac{1}{\rho c_p} \frac{dF(z)}{dz} \), and we have all of the quantities in this equation. Plugging into the heating rate equation, we get

\[
\frac{dT}{dt} = \frac{-1}{0.5 \text{kg m}^{-1} 1004 \text{J kg}^{-1} \text{K}^{-1}} \left( \frac{600 \text{ W m}^{-2} + 200 \text{ W m}^{-2} - 1200 \text{ W m}^{-2}}{12,000 \text{ m} - 11,000 \text{ m}} - \frac{200 \text{ W m}^{-2} + 320 \text{ W m}^{-2} - 300 \text{ W m}^{-2} - 250 \text{ W m}^{-2}}{12,000 \text{ m} - 11,000 \text{ m}} \right)
\]

\[
\frac{dT}{dt} = \frac{-1}{0.5 \text{kg m}^{-1} 1004 \text{J kg}^{-1} \text{K}^{-1}} \left( \frac{-400 \text{ W m}^{-2}}{12,000 \text{ m} - 11,000 \text{ m}} - \frac{-30 \text{ W m}^{-2}}{12,000 \text{ m} - 11,000 \text{ m}} \right) = 7.4 \times 10^{-4} \text{ K s}^{-1}
\]

\[
\frac{dT}{dt} = 7.4 \times 10^{-4} \text{ °C s}^{-1} \cdot \frac{3600 \text{ s}}{\text{hr}} = 2.65 \text{ °C hr}^{-1}
\]

14. How long is the day in Denver on January 1?

To compute the length of day, we first need to compute the hour angle \( H \) when the SZA \( \theta = 90 \)

\[ H = \cos^{-1}(-\tan \phi \tan \delta) \]

The latitude \( \phi \) of Denver is 39°. We need the declination angle, \( \delta \), on Jan 1.

\[ \delta = -23.5 \cos \left( \frac{360}{365} (D_a + 10) \right) = \delta = -23.5 \cos \left( \frac{360}{365} (1+10) \right) = -23.08° \]
From this, \( H \) is
\[
H = \cos^{-1}(\tan(-39^\circ)\tan(-23.08^\circ)) = 70^\circ
\]

Therefore, the day length is \( 2 \times 70^\circ / (15 \text{ hr}^{-1}) = 9.33 \text{ hours} \)

15. Mars has a planetary albedo of 0.25, and an effective emission temperature of 210 K. Because of the lack of an appreciable atmosphere, the emission temperature and surface temperature are almost the same. One idea put forth for making Mars habitable is to cover it with black cyanobacteria that would absorb sunlight, thus raising the planet’s temperature. Assuming one could make Mars purely black, what would the new temperature be?

We need to solve the equation for emission temperature of a planetary body,

\[
T_e^4 = \frac{S_0(1-\alpha)}{4\sigma}
\]

If Mars were to be purely black, \( \alpha=0 \). However, we don’t know the appropriate \( S_0 \) for Mars. However, given the data above (\( \alpha \) and \( T_e \)), we can calculate \( S_0 \) for Mars.

\[
S_0 = \frac{4\sigma T_e^4}{(1-\alpha)} = \frac{4 \times 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \times (210 \text{ K})^4}{1-0.25} = 588 \text{ W m}^{-2}
\]

Plugging this into our equation for temperature, but with the new albedo

\[
T_e = \left( \frac{S_0(1-\alpha)}{4\sigma} \right)^{\frac{1}{4}} = \left( \frac{588 \text{ W m}^{-2} \times 1}{4 \times 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}} \right)^{\frac{1}{4}} = 226 \text{ K}
\]

**EQUATIONS**

\[
\begin{align*}
\cos \theta &= \sin \phi \sin \delta + \cos \phi \cos \delta \cos \tau \\
T_e^4 &= S(1-\alpha)/4\sigma \\
\tau &= (Z - 12) \times 15^\circ + F / 4 \\
F(T) &= \sigma T^4 \\
\cos H &= -\tan \phi \tan \delta \text{ (for sunrise & sunset)} \\
F &= S(d_m/d)^2 \\
\tau &= \int k \rho dz = \int Q_{\text{ext}} \sigma N dz \\
F &= F_0 e^{-\gamma/\mu} \\
\frac{dT}{dt} &= -1 PC_p \frac{dz}{dz} \\
\lambda_{\text{max}} &= \frac{2897 \mu mK}{T} \\
\delta &= -23.5 \cos \left\{ \frac{360}{365} \left( D_n + 10 \right) \right\}
\end{align*}
\]

**CONSTANTS**

\[d_m = 149.6 \times 10^6 \text{ km}\]
\[r_{\text{earth}} = 6378 \text{ km}\]
\[c_p = 1004 \text{ J kg}^{-1} \text{ K}^{-1}\]
\[\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}\]
\[g = 9.8 \text{ m s}^{-2}\]
\[S_0 = 1360 \text{ W m}^{-2}\]
\[h = 6.63 \times 10^{-34} \text{ J s}\]
\[c = 2.99 \times 10^8 \text{ m s}^{-1}\]
\[k = 1.38 \times 10^{-23} \text{ J K}^{-1}\]