Read

Bohren and Clothiaux Ch.1; Ch 4.1-4.2
Thomas and Stamnes, Ch. 2.1-2.6; 4.3.1-4.3.2

Radiative Transfer Applications

What is it good for?
RT is a key part of remote sensing and climate modeling.

Remote sensing: retrieving atmosphere or surface properties from measured radiances.

Climate: radiative heating affects temperature, and hence atmosphere dynamics, which affect the atmospheric state.

What is Radiation?

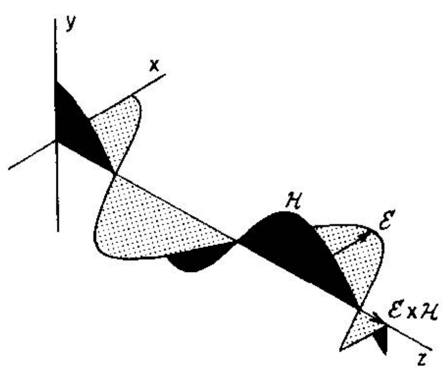
Classically, radiation is oscillating electric and magnetic fields.

E field perpendicular to propagation direction.

E field is a two component vector: the relative amplitudes and phases of the E field components give rise to polarization.

Intensity (or energy) is proportional to square of electric field.

Radiation properties: Intensity, Phase, Polarization.
Radiation depends on frequency, space, time, direction.



A schematic view of an electromagnetic wave propagating along the z axis. From Maxwell's equations the E and M fields are 90◦ out of phase and perpendicular to each other and the direction of travel.

The Spectrum

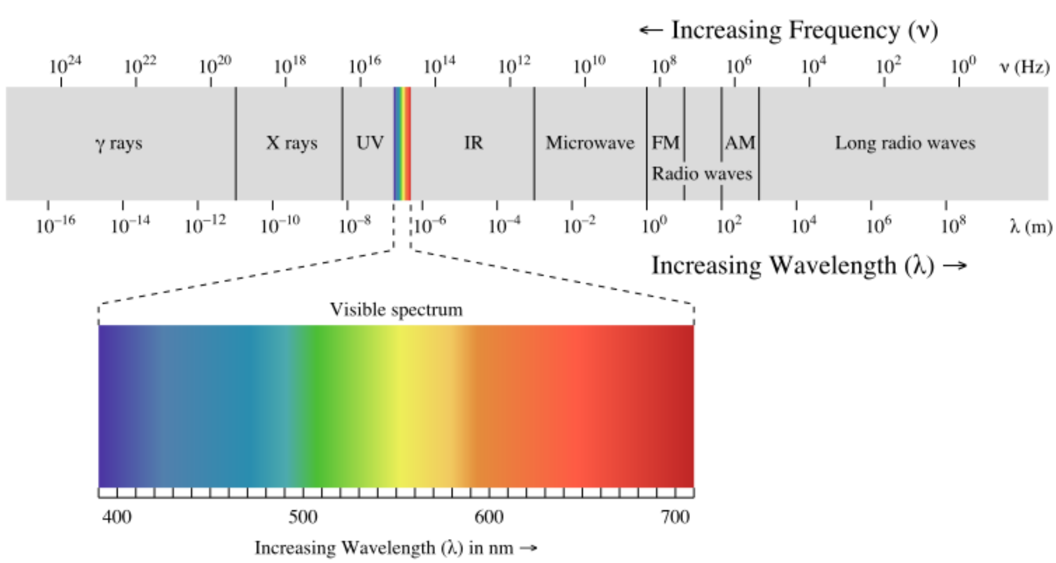
Wavelength, λ, related to frequency, ν, and speed of light c (2.998 m/s): $\lambda\nu = c$

Wavenumber, $\tilde{\nu}$, is number of waves in a given length (usually cm) and is proportional to frequency:

$$\tilde{\nu} = \frac{1}{\lambda} = \frac{10000\,\text{cm}^{-1}\,\mu\text{m}}{\lambda},\ \lambda \text{ in units of } \mu\text{m}.$$

Example: 8-12 μm atmospheric window is 833-1250 cm^{-1}

Wavelength range (μm)	Spectral region
0.01 -0.38	Ultraviolet
0.38 -0.75	Visible
0.75 -4	Near infrared
4 -20	mid infrared
20 -1000	far infrared
>1000	microwave
< ~4.0	Shortwave (from Sun = solar)
> ~4.0	Longwave (from Earth = terrestrial)



Radiation: is it a wave or a particle?

Light (electromagnetic radiation) has a dual nature. In some experiments it behaves like waves; in others it behaves like streaming particles (quanta) called photons.

The energy of a photon is:

$$E = h\nu = hc/\lambda = hc\tilde{\nu}$$

h is Planck's constant (h = 6.626 × 10^{-34} J s)

Which should we use? Depends on the problem.

Solid Angle

Angle: $\varphi = s/r$ where s is length of arc, r is radius.

Solid angle is two-dimensional analog: $\Omega = A/r^2$. Solid angle defines a measure of the set of all directions from the origin to area A.

The solid angle subtended by a surface S is defined as the area of the surface's projection onto a unit sphere:

$$\Omega = \iint \frac{\hat{\mathbf{n}} \cdot d\mathbf{a}}{r^2}$$

where $\hat{\mathbf{n}}$ is a unit vector from the origin, $d\mathbf{a}$ is the differential area of a surface, and r is the distance from the origin to the surface.

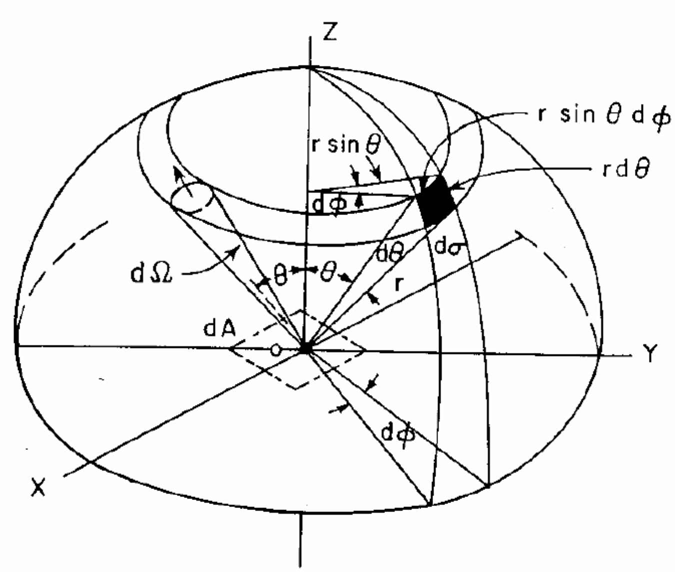
In spherical coordinates with polar angle θ and azimuth angle φ, this becomes: $\Omega = \iint \sin\theta d\theta d\varphi = 4\pi$ if integrated over entire sphere.

Solid angle of sun (angular width, α, is 0.5°):

$$\Omega_{sun} = 2\pi \int_{\cos(\vartheta_s/2)}^{1} d\mu \approx 6\times10^{-5} \text{ sr}$$

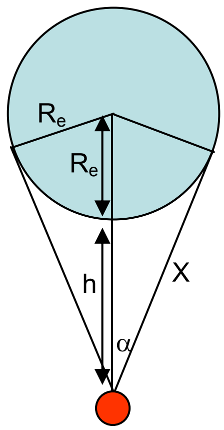
Because $R_{S-E} >> R_{Sun}$ wecould have calculated

$$\Omega_{Sun} = A_{Sun}\Big/R_{S-E}^2 = \frac{\pi(\frac{\alpha}{2}R_{S-E})^2}{R_{S-E}^2} = \pi\alpha^2\Big/4$$



Problem:

A satellite orbits the Earth at an altitude h above the surface. What solid angle does the Earth subtend at the orbit of the satellite?



$$\Omega = \int_0^{2\pi}\int_0^{\alpha} \sin\theta d\theta d\phi = 2\pi \int_{\cos\alpha}^{1} d\mu = 2\pi(1-\cos\alpha)$$

$$\cos\alpha = \frac{\sqrt{R_e^2 + h^2 + 2h R_e - R_e^2}}{h + R_e}$$

$$\Omega = 2\pi\left[1 - \sqrt{h^2 + 2h R_e}\Big/(h + R_e)\right]$$

Definitions

Monochromatic radiance (occasionally called intensity):

Rate of transfer of radiant energy per unit wavelength interval per unit solid angle through a unit area perpendicular to the direction of propagation.

$$L_\lambda = \frac{dE}{dt dA_\perp d\lambda d\Omega}$$ Units: W m^{-2} nm^{-1} sr^{-1}

Monochromatic irradiance, or flux density:

Rate of transfer of radiant energy per unit wavelength interval through a unit area.

$$F_\lambda = \frac{dE}{dt dA d\lambda}$$ Units: W m^{-2} nm^{-1}

$F_\lambda = L_\lambda \cos\psi d\Omega$; $dA_\perp = \cos\psi\ dA$, where ψ is the angle between the normal to area dA and the direction of incidence.

If dA lies in the x,y plane and is centered at the origin, in spherical coordinates $\theta = \psi$ and: $F_\lambda = \int_\Omega L_\lambda(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$

For isotropic radiation (L independent of direction) the total downward irradiance is given by the integral is over the hemisphere:

$$F_\lambda = \int_0^{2\pi}\int_0^{\pi/2} L_\lambda(\theta,\phi)\cos\theta\sin\theta d\theta d\phi = 2\pi L_\lambda \int_0^1 \mu d\mu = \pi L_\lambda$$

Emission: the birth of photons

All matter (gas, liquid, or solid) at *all* temperatures emits radiation at *all* frequencies at *all* times (even at 0 K?)

Emitted radiation is not stimulated by an external source of radiation (that is *scattered radiation*)

Planck Function

Consider a container held at fixed T, with walls so thick that no photons escape. At equilibrium, the distribution of photon energies is determined solely by this temperature. Radiation in equilibrium with matter is ***blackbody radiation.*** Blackbody radiation is unpolarized and isotropic.

Blackbody radiation requires the existence of a blackbody: a blackbody cannot be excited to radiate by an external source of radiation more than it would in isolation

The energy distribution function (spectral distribution) of the photons in the container is given by the Planck distribution: $P(\omega) = \frac{\hbar\omega}{4\pi c^2}\frac{1}{\exp(\hbar\omega/k_B T)-1}$

k_B is Boltzmann's constant, 1.38×10^{-23} J K^{-1}.

Note: $P(\omega)$ is in units of *irradiance*. Since blackbody radiation is isotropic, if $B(\omega)$ is the Planck radiance, then $P(\omega) = \pi B(\omega)$.

Note that the equation above is used in Bohren and Clothiaux, and includes circular frequency ($\omega = 2\pi\upsilon$) and $\hbar$ (= $h/2\pi$). You may more frequently encounter Planck radiance and *wavenumber* as the independent variable. Make sure you recognize the various relationships.

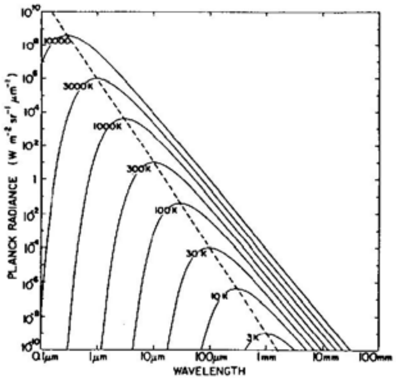
Change of Variables

The physical content of the Planck distribution (or any distribution function) is defined by its integral properties: $P(\lambda) = P\{\omega(\lambda)\}\left|\frac{d\omega}{d\lambda}\right| = P\{\omega(\lambda)\}\frac{2\pi c}{\lambda^2}$

Wien's displacement law

How is the Planck function displaced as T is increased?

$$\lambda_{max} T = 2898\ \mu\text{m K}$$



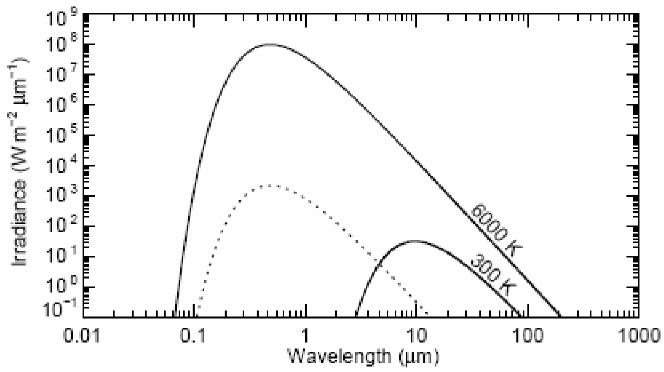
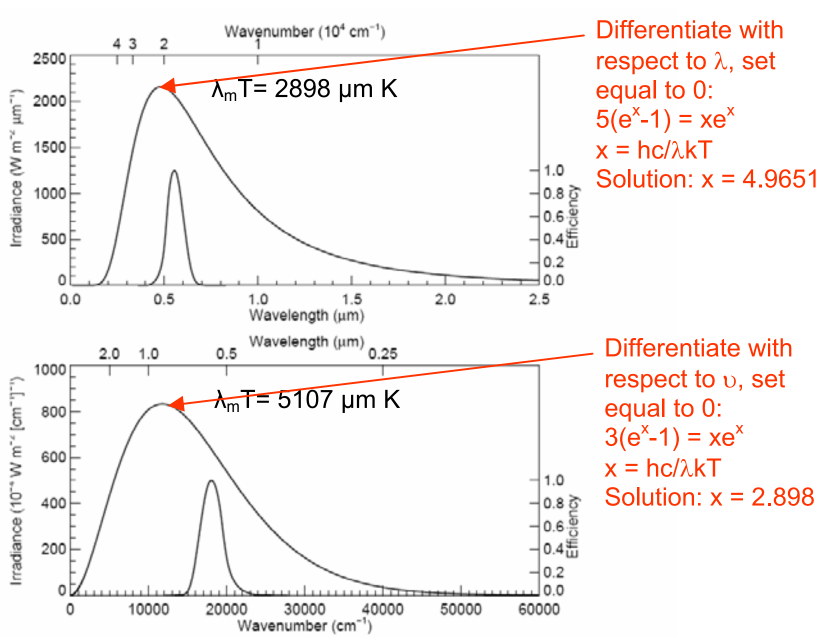


Figure 1.4: Planck function for 6000 K and 300 K. The dashed curve is the irradiance at the top of the atmosphere from a 6000 K blackbody at the Earth-sun distance, which approximates the solar irradiance.

Displacement laws depend upon the independent variable:



Stefan-Boltzmann Law

Spectrally integrate the Planck distribution to determine total irradiance from blackbody:

$$F = \int_0^\infty \frac{\hbar\omega}{4\pi c^2} \frac{1}{\exp(\hbar\omega / k_B T) - 1} d\omega = \sigma T^4$$

σ is the Stefan-Boltzman constant, 5.699×10^{-8} W m^{-2} K^{-4}.

Emissivity

Emissivity is the fraction of emitted radiance to that from a blackbody:

$$L_\lambda = \varepsilon_\lambda B_\lambda(T)$$

Kirchoff's law: emissivity = absorptivity:

$$\varepsilon_\lambda = a_\lambda$$

What is ε_λ and a_λ for a blackbody?

Brightness and color temperature

What temperature must a blackbody have in order to match emitted radiation (over entire spectrum or a selected bandpass)?

Consider the integral of the Planck function Eq. over any range of frequencies:

$$\int_{\omega_1}^{\omega_2} \frac{\hbar\omega}{4\pi c^2} \frac{1}{\exp(\hbar\omega / k_B T) - 1} d\omega$$

This integral approaches *0* as $T \rightarrow 0$ and *1* as $T \rightarrow \infty$, and its derivative with respect to T is always positive. Thus, for any emitted radiation we can always find one and only one temperature such that the equation above matches it. This is ***brightness temperature***.

The ***color temperature*** of a source of visible radiation is the temperature of a blackbody with the same perceived color. Blackbodies of all temperatures can match only a small sample of possible colors.

Color temperature of average daylight (sunlight plus skylight) is about 6500 K.
What would be the color temperature of the blue sky?

Limiting forms of the Planck distribution

Consider Planck distribution as a function of wavelength:

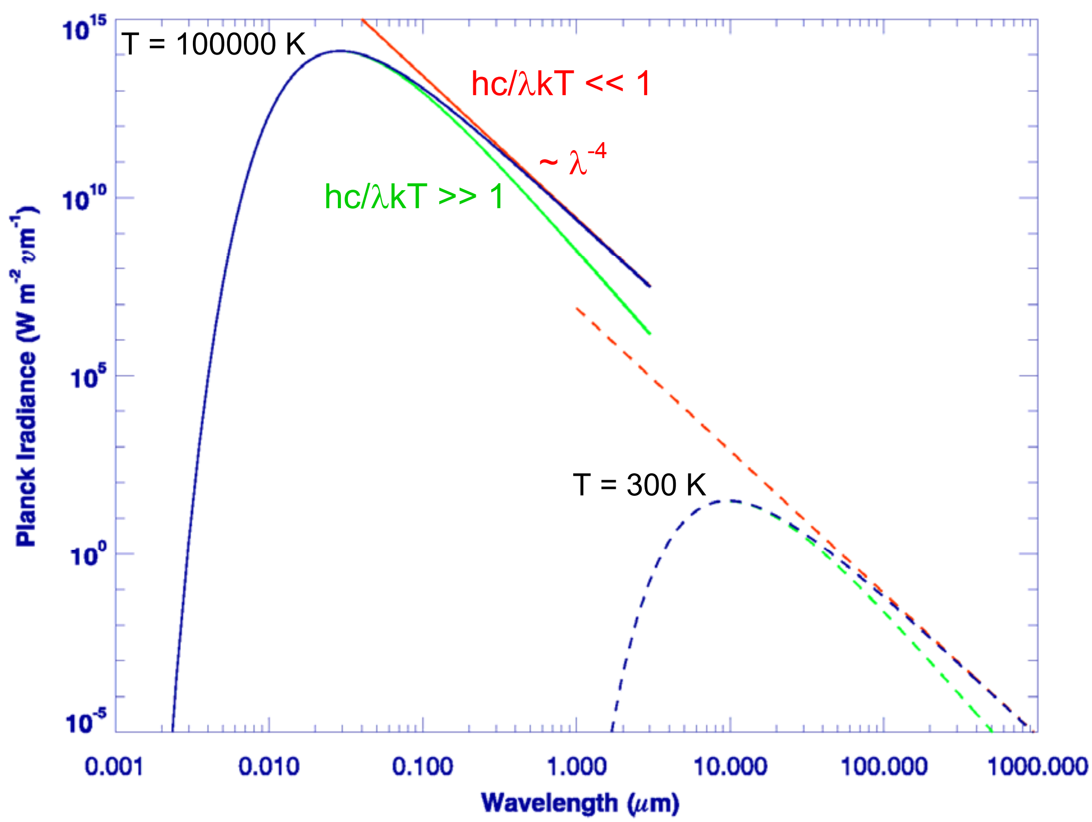
$$P(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

Wien's limit : $hc/\lambda k_B T >> 1$ (short wavelength/low T)

$$P(\lambda) = \frac{2\pi hc^2}{\lambda^5} \exp(-hc/\lambda k_B T)$$

Rayleigh-Jeans: $hc/\lambda k_B T << 1$ (long wavelength/high T)

$$P(\lambda) = \frac{2\pi ck_B T}{\lambda^4}$$



Problem

The emissivity of quartz at 9.5 μm is about 0.5. Calculate the irradiance at 9.5 μm of quartz at 230 K. Then calculate the brightness temperature.

How does this compare to the thermodynamic temperature? For what wavelengths would the brightness temperature for an emissivity of 0.5 actually be half the thermodynamic temperature? *Hint: recall limiting forms of the Planck distribution*

Local Thermodynamic Equilibrium

In local thermodynamic equilibrium (LTE) emission depends only on temperature and absorption properties of matter, not on the radiation field itself.

In LTE:

1. The time between quantum transitions from collisions << time between transitions from radiation.
2. Boltzmann distributions apply to relevant atomic/molecular levels.
3. Applies up to 50 to 80 km in Earth's atmosphere (depends on υ).

LTE applies in the Earth's troposphere and stratosphere, but not the mesosphere and thermosphere.

Thomas & Stamnes (p. 102) discuss LTE and also consider non-LTE, where the radiation field and population of quantum states are coupled. However, we will always assume LTE in this class.

More on Radiative Quantities/Definitions

***Invariance* of radiance**: In the absence of extinction radiance is *invariant* with distance r from a source.

For small solid angle: $F = L\Omega\cos\theta$ or $L = F/\Omega\cos\theta$.
F is decreasing by $1/r^2$ from source but Ω is also decreasing by $1/r^2$.

Two properties of irradiance:

1. The upward and downward irradiances depend on the orientation of the reference plane dividing the set of all directions into upward and downward hemispheres.

2. Although radiance determines irradiance, the converse, in general, is not true: if we know the irradiance, we cannot uniquely determine the corresponding radiance.

Monochromatic *net* irradiance (or flux) is the integral of the normal component of radiance over all solid angle: $F_\lambda = \int_{4\pi} L_\lambda(\theta,\phi)\cos\theta d\Omega$

Net irradiance (flux) for a horizontal plane is the difference in upwelling and downwelling hemispheric irradiance, or the net radiative energy flow in an atmosphere:

$$F_{net,\lambda} = F_\lambda^{\uparrow} - F_\lambda^{\downarrow} = \int_0^{2\pi}\int_{-1}^{1} L_\lambda(\mu,\varphi)\mu d\mu d\varphi$$

Actinic flux, or **average intensity**, is the total spectral energy at point (used in photochemistry): $A_\lambda = \int_{4\pi} L_\lambda(\theta,\phi)d\Omega = 4\pi \overline{L}_\lambda$ where $\overline{L}_\lambda$ is the mean radiance.

Problem (B&C 1.29)

The wavelength at which the Planck function has a maximum (for a given temperature) is not an absolute quantity but depends on the independent variable chosen, which is arbitrary. However, there is a peak wavelength that can be defined unambiguously as a median wavelength: the wavelength that divides the Planck function into two regions of equal area. First, convince yourself that this median wavelength is independent of the variable in the Planck function and why. Then determine the displacement law for the median wavelength. What is the median wavelength for a 6000 K blackbody?

Problem (B&C 1.27)

What is the temperature of a blackbody with an emission spectrum that peaks at the same wavelength as the solar spectrum (outside Earth's atmosphere)? Suppose that the sun were replaced by a blackbody of the same radius and distance to Earth. What is the temperature of this blackbody such that the solar irradiance is its presently accepted value (about 1361 W m^{-2})? The angular width of the sun is about 0.5°.