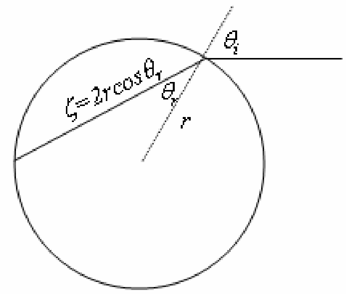
Geometric Optics approximation for single scattering albedo in the limit of weak absorption



Consider a sphere of radius *a*. A ray entering the sphere at an angle of θ_i to the normal to the surface traverses a path $\zeta = 2r\cos\theta_r$ before exiting or undergoing an internal reflection. θ_i .and θ_i are related through Snel's law: $m\sin\theta_i = \sin\theta_r$ where *m* is the refractive index. Let *t* be the fraction of incident radiation penetrating the sphere and *r* be the fraction of internally incident radiation internally reflected. The fractional absorption over a distance ζ is $1-\exp(-\kappa\zeta)$ where κ is the absorption coefficient of water. If reflected, it traverses another distance ζ, and so on. Therefore, the total fraction of the radiation incident from this direction that is absorbed is:

$$t(1-\exp(-\kappa\zeta)) + r\exp(-\kappa\zeta)t(1-\exp(-\kappa\zeta)) + (r\exp(-\kappa\zeta))^2 t(1-\exp(-\kappa\zeta)) + \ldots =$$

$$t(1-\exp(-\kappa\zeta))\{1 + (r\exp(-\kappa\zeta)) + (r\exp(-\kappa\zeta))^2 (r\exp(-\kappa\zeta))^3 + \ldots\} = \frac{t(1-\exp(-\kappa\zeta))}{1-r\exp(-\kappa\zeta)}$$

For weak absorption $\kappa\zeta << 1$ and from Fresnel equations $t = 1 - r$. Substituting, fractional absorption becomes $\dfrac{t(1-\exp(-\kappa\zeta))}{1-r\exp(-\kappa\zeta)} \approx \dfrac{t\kappa\zeta}{1-r} = \kappa\zeta = \kappa 2r\cos\theta_r$.

To get the absorption cross section we must integrate all incident angles over the hemisphere:

$$C_{abs} = \int_0^{2\pi}\int_0^{\pi/2} \kappa 2r\cos\theta_r r^2 \cos\theta_i \sin\theta_i d\theta_i d\vartheta = 4\pi r^3\kappa \int_0^{\pi/2}\left(1-\frac{\sin^2\theta_i}{m^2}\right)\cos\theta_i \sin\theta_i d\theta =$$

$$\frac{4\pi}{3} r^3 \kappa m^2 \left[1-(1-m^{-2})^{3/2}\right] \text{(using Snel's law).}$$

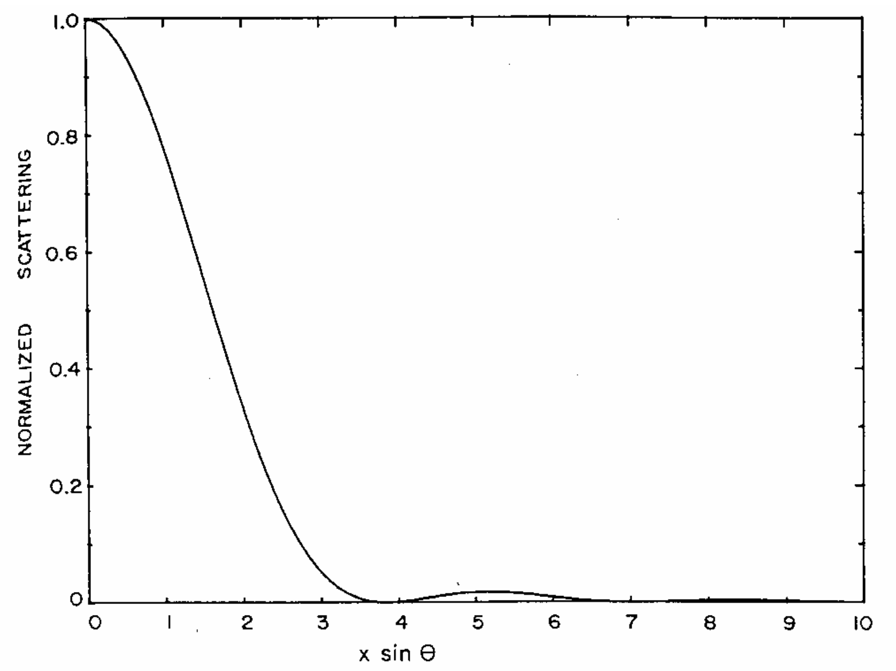
$$1-\varpi_0 = \frac{C_{abs}}{C_{sca}} = \frac{2}{3} r\kappa m^2 \left[1-(1-m^{-2})^{3/2}\right]$$ using the geometric optics approximation $C_{sca} = 2\pi r^2$. For *m* = 1.33, we get $1-\varpi_0 = 0.84r\kappa$

Problem:

A corona is a colored ring of light around the moon or Sun when viewed through a thin cloud. The outer part of the disk often has a brownish-red tinge. If the diameter of the disk is 10 times that of the moon, what is the approximate radius of particles in the cloud?

Check out http://www.atoptics.co.uk/droplets/corim1.htm for images of the corona.

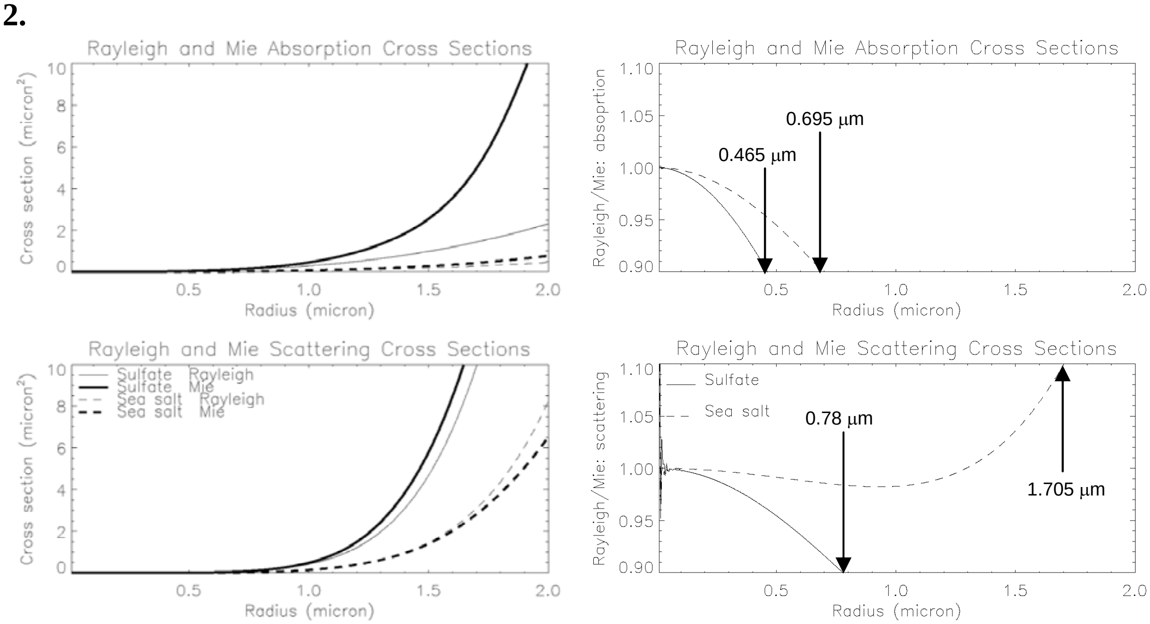
Recall the scattering diagram for a circular disk.



The forward lobe is contained within $x\sin\Theta \approx 3$. Recall, this is geometric optics so x >> 1 and therefore, $\sin\Theta \ll 1$ so sin $\sin\Theta \approx \Theta$ and so the edge of the primary lobe is at $\Theta \approx 3/x$ or $\Theta \approx 170°/x$. If the diffraction ring is ten times the diameter of the moon, then $\Theta = \frac{10 \cdot 0.5^\circ}{2} = 2.5^\circ$ so $x \approx \frac{170^\circ}{2.5^\circ} = 68$ and $r = \frac{68\lambda}{2\pi}$. Using 0.65 μm for red light, r ≈ 7 μm. This is appropriate for liquid water droplet clouds.

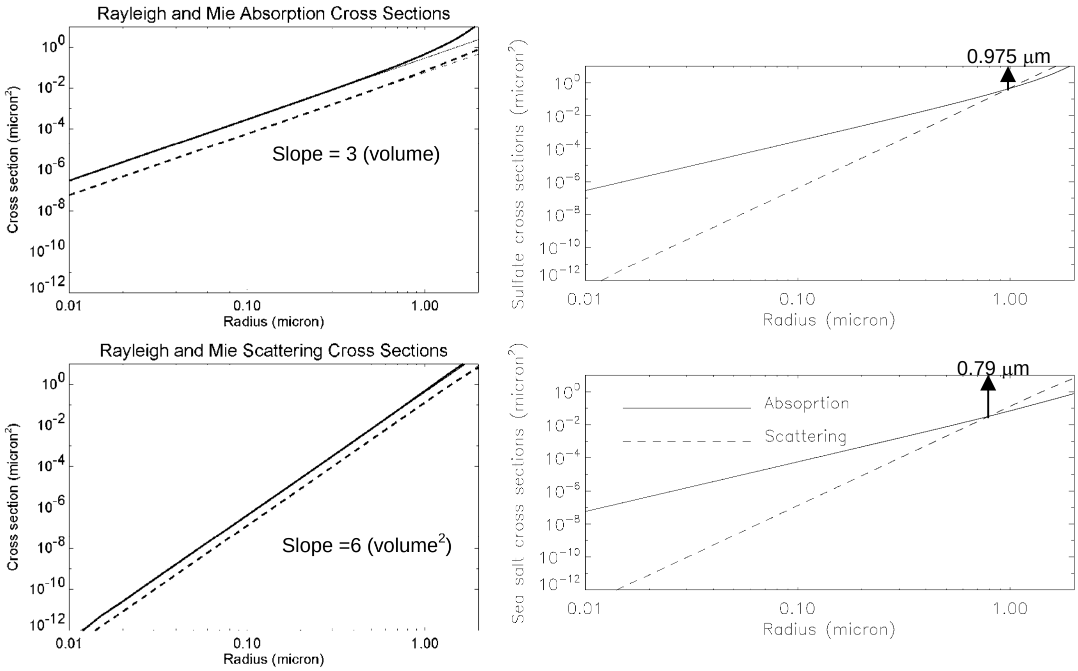
Solutions to Lab 2:

2.



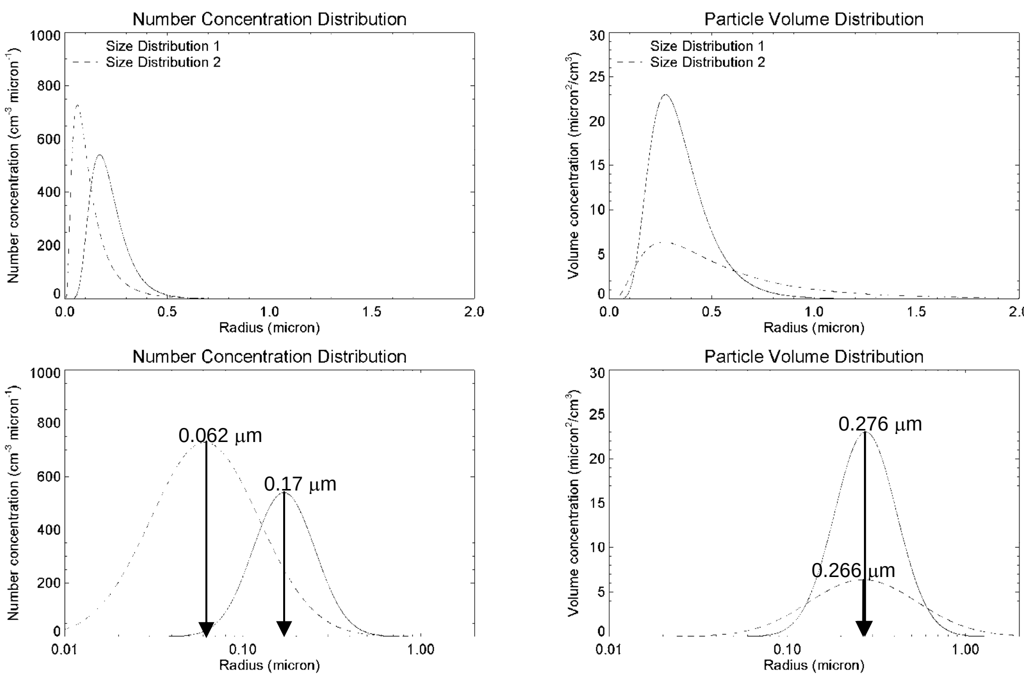
C_{abs} proportional to v; C_{sca} to v^2. The aerosol with the smaller index of refraction stays in the Rayleigh regime longer because the size of the Mie coefficients depends on the index of refraction.

3.



There is more absorption for ammonium sulfate so it takes longer (larger radius) for scattering to “catch up”.

4.



Even though the second distribution has half the r_0, the volume distribution peaks are almost the same. This is because the smaller r_0 distribution is also wider (larger) so there are more large particle which contribute more to the volume.

1. Code the expressions for Rayleigh absorption and scattering cross sections:

```
m1=complex(2.190,0.130) ; sulfate
m2=complex(1.540,0.015) ; sea salt
Csca1 = (128/3.)*pi^5 * (r^6/lambda^4) * abs((m1^2-1)/(m1^2+2))^2
Cabs1 = 8*pi^2 * r^3/lambda * imaginary((m1^2-1)/(m1^2+2))
Csca2 = (128/3.)*pi^5 * (r^6/lambda^4) * abs((m2^2-1)/(m2^2+2))^2
Cabs2 = 8*pi^2 * r^3/lambda * imaginary((m2^2-1)/(m2^2+2))
```

4. Number and volume distributions

```
n_r = N/(sqrt(2*!pi)*sigma) * (1/r) * exp(-alog(r/r0)^2 /(2*sigma^2))
vol_dist = (4*pi/3.)*r^3* lognormal_distribution(r, Ntot, r0, sigma)
```

5. Volume absoprtion coefficient:

```
Cabs1 = 8*pi^2 * r^3/lambda * imaginary((m1^2-1)/(m1^2+2))
Cabs2 = 8*pi^2 * r^3/lambda * imaginary((m2^2-1)/(m2^2+2))
print, total(Cabs1*dist1)*delta_r
print, total(Cabs2*dist2)*delta_r
delta_r = radmax/Nrad
```

Don't forget the Δr!

$\beta_{abs} = 4.8 \times 10^{-4}$ km^{-1} for the sulfate aerosol, size distribution 1
$\beta_{abs} = 5.1 \times 10^{-5}$ km^{-1} for the sea salt aerosol, size distribution 2

Most aerosol particles in the mid-infrared have very low optical depths.