ATOC/ASTR notes 9/10/07

Read:

- Bohren and Clothiaux Ch.3.1-3.2, 3.5
- Supplemental reading (to be e-mailed/put on web)

Problem

What is the temperature of a blackbody with an emission spectrum that peaks at the same wavelength as the solar spectrum (outside Earth's atmosphere)? Suppose that the sun were replaced by a blackbody of the same radius and distance to Earth. What is the temperature of this blackbody such that the solar irradiance is its presently accepted value (about 1361 W m^{-2})? The angular width of the sun is about 0.5°.

a. Wien's law: $Tb = \dfrac{2898}{\lambda_{\max}(\mu m)} K = 6037K$ for $\lambda_{\max} = 0.48 \mu m$ K

b. $\sigma T^4 = F_o \left(\dfrac{R_{S-E}}{r_S} \right)^2$ where r_S and R_{S-E} are radius of sun and Sun-Earth distance respectively. Angular width of sun is 0.5º so tan(0.25º) = $(r_S/R_{S-E}) \approx 0.25 * \pi/180$.

So $T = \left[\dfrac{1361}{5.67 \times 10^{-8}} \left(\dfrac{180}{0.25\pi} \right)^2 \right]^{1/4} = 5959K$

Net Irradiance

Net irradiance is the integral of the normal component of radiance over all solid angles:

$$F_{net,\lambda} = F_\lambda^{\uparrow} - F_\lambda^{\downarrow} = \int_{4\pi} L_\lambda \cos\theta d\Omega = \int_0^{2\pi}\int_{-1}^{1} L_\lambda(\mu,\varphi)\mu d\mu d\varphi$$

Actinic flux

Actinic flux, or average intensity, is the total spectral energy at point (used in photochemistry):

$$A_\lambda = \int_{4\pi} L_\lambda(\theta,\phi) d\Omega = 4\pi \overline{L}_\lambda \text{ where } \overline{L}_\lambda = \frac{\int_{4\pi} L_\lambda(\theta,\phi) d\Omega}{\int_{4\pi} d\Omega}$$

Local Thermodynamic Equilibrium

Kirchoff's law only applies to systems in ***Local Thermodynamic Equilibrium*** (LTE):

- **LTE**: the time between quantum transitions due to collisions is much less than the time between transitions due to absorption of radiation.
- Molecules exchange energy with one another more rapidly than they do with the radiation field or other sources of energy
- Applies from the surface up to 50 to 80 km (mesosphere/thermosphere) in Earth's atmosphere, depending on wavelength.
- Emission depends only on temperature and absorption properties of matter, not on the radiation field itself.
- The time between quantum transitions due to collisions << time between transitions due to absorption of radiation.
- Boltzmann distributions apply to relevant atomic/molecular energy levels.
- LTE also breaks down in lasers, fluorescent bulbs, discharge tubes, light emitting diodes, where average molecular energy levels are "pumped up" by various means beyond levels expected due to the thermodynamic temperature of the molecules.
- Emissions in non-LTE systems can be far greater than from a blackbody
- Thomas & Stamnes (p. 102) discuss LTE and also consider non-LTE, where the radiation field and population of quantum states are coupled. However, we will always assume LTE in this class.

Extinction Law

The reduction in radiance over pathlength *ds* is proportional to the incident radiance:

$$dI_\lambda = -\beta_\lambda I_\lambda ds$$

β_λ: extinction coefficient [length^{-1}]

$\beta_\lambda = \kappa_\lambda \rho$

κ_λ : mass extinction coefficient [length^2 mass^{-1}]

$\beta_\lambda = N\sigma_\lambda$

σ_λ: extinction cross section [length^2]; N is number density

$$dI_\lambda = -\beta_\lambda I_\lambda ds = -I_\lambda d\tau_\lambda$$

τ: optical path; dimensionless depth of layer, measure of strength and number of optically active path of particles along a beam.

$$\tau_\lambda = \int \beta_\lambda\, ds = \int \kappa_\lambda \rho ds = \int N_\lambda \sigma_\lambda ds$$

Extinction (e) = scattering (s) + absorption (a):

$\beta_e = \beta_s + \beta_a$

$\kappa_e = \kappa_s + \kappa_a$

$\sigma_e = \sigma_s + \sigma_a$

$\tau_e = \tau_s + \tau_a$

Optical depth is optical path in vertical from top down:

$$\tau(z) = \int_z^\infty \kappa(z)\rho(z)dz; \tau(s) = \tau(z)/\mu$$

Absorber (or scatterer) Amount

If the mass extinction coefficient κ_λ is uniform, then the optical depth is $\tau_\lambda = \kappa_\lambda u$

u is the integrated *absorber amount* between heights z_1 and z_2 (e.g. g/cm^2):

$$u = \int_{z_1}^{z_2} \rho_a(z)dz = \frac{1}{g}\int_{p(z2)}^{p(z1)} q_a dp \text{ where } q_a = \rho_a / \rho_{air}$$

q_a is the mass mixing ratio.

For well mixed gases (q_a constant) the absorber amount is proportional to pressure difference across layer.

For more than one absorbing (or scattering) gas (or particle) , the optical paths add:

$$\tau_\lambda = \Sigma \kappa_{i,\lambda} u_i$$

Beer-Lambert Law

Extinction Law (equation of transfer with no sources):

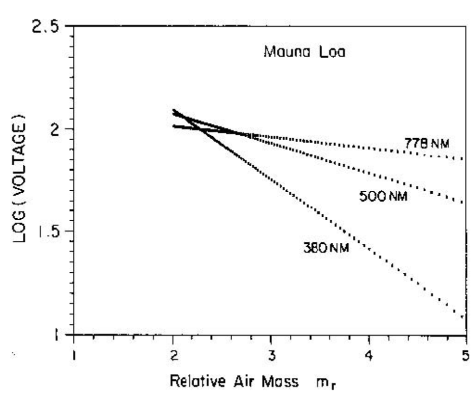
$$dI_\lambda = -\beta_\lambda I_\lambda ds = -I_\lambda d\tau_\lambda$$

Integrate: $I_\lambda(s) = I_\lambda(0)\, exp(-\tau_\lambda(s))$

Langley Plot

Consider the direct solar irradiance incident from the direction $\mu_0 = \cos(\theta_0)$:

$$F_\lambda = F_{0,\lambda}\, exp(-\tau_\lambda/\mu_0)$$
$$\ln F_\lambda = \ln F_{0,\lambda} - \tau_\lambda(1/\mu_0) = \ln F_{0,\lambda} - m\tau_\lambda$$



Define ***airmass***, $m = 1/\mu_0 = \sec(\theta_0)$
Plot ln F_λ vs m: slope τ_λ, intercept ln$F_{0,\lambda}$

Used for calibration; measurement of τ_λ

Homework:

A sunphotometer has a silicon detector that is highly linear in measuring the direct solar flux. When the solar zenith angle is 60° *the sunphotometer measures a voltage, V*, of 1.673 *in the 870 nm channel. When calibrated with the Langley plot method, the voltage extrapolated to the top of the atmosphere is* V_0 = 2.127. *The effect of molecular absorption is negligible at this wavelength, but the molecular scattering optical depth is 0.015.*

a) Calculate the aerosol optical depth at this wavelength.
b) Lidar backscattering measurements show that most of the aerosols are in the 1.5 km thick boundary layer. What is the average volume extinction coefficient?
c) A typical mass extinction coefficient for sulfate aerosols at this wavelength is κ = 2.5 m^2 /g. *What is an estimate for the aerosol loading* (g/m^2)?

Light Scattering by Particles

- Scattering is the redirection of light: reflection, refraction, diffraction, etc., are all forms of scattering.
- Radiation can be attenuated by absorption or scattering or both.
- Amount of scattering by a distribution of particles is usually quantified with the volume scattering coeffcient:

β_{sca} , the fraction of radiation scattered per distance beam travels

- Particles attenuate by both absorption and scattering:

$$\beta_{ext} = \beta_{sca} + \beta_{abs}$$

- The relative amount of scattering to scattering plus absorption (extinction) is given by the single scattering albedo:

$$\varpi_o = \beta_{sca}/(\beta_{abs} + \beta_{sca}) = \beta_{sca} / \beta_{ext}$$

- Pure scattering: $\varpi_o = 1$ (conservative scattering)
- Pure absorption: $\varpi_o = 0$

Phase Functions

- The directional distribution of scattered radiation is the *phase function*: $P(\Theta)$ is the fraction scattered in direction Θ.
- Θ is the scattering angle: $\Theta = 0°$ is forward scattering, $\Theta = 180°$ is backscattering.
- Isotropic scattering (equal in all directions): $P(\Theta) = 1$

Normalization: $\int_0^{2\pi} \int_{-1}^{1} P(\Theta) d(\cos\Theta) d\varphi = 4\pi$

Asymmetry parameter

Asymmetry parameter is the first moment (average cosine) of phase function:

$$g = \frac{1}{2}\int_{-1}^{1} P(\cos\Theta)\cos\Theta d(\cos\Theta)$$

- Equal forward and backward $g = 0$
- Totally forward scattering $g = 1$
- Totally backward scattering $g = -1$
- For water clouds in visible, $g \sim 0.85$

Scattering Cross Section

Extinction, scattering, and absorption for single particles is measured in cross sectional area: $C_{ext} = C_{sca} + C_{abs}$

Volume extinction coefficient related to cross section by $\beta_{ext} = NC_{ext}$ (N is number density)

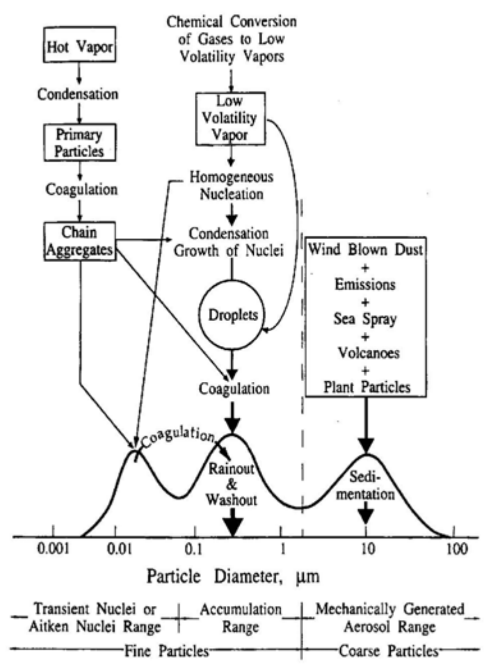
Scattering and absorption efficiency: unitless ratio of cross section to projected area of particle. For sphere:

$$Q_{sca} = C_{sca}/\pi r^2 \qquad Q_{abs} = C_{abs}/\pi r^2 \qquad Q_{ext} = C_{ext}/\pi r^2$$

Aerosols Particles (remember, aerosol refers to particles plus carrier gas):

- Small particles (< 10 μm), solid or liquid
- Sulfates (sulfuric acid, ammonium sulfate), hydrated depending on RH. From gas-to-particle conversion of SO_2, DMS.
- Mineral -wind blown dust from soils (quartz, iron oxide) (large size).
- Sea salt -from air bubbles bursting at sea surface (large).
- Combustion -from fossil fuels, forest burning (small).

-Aitken nuclei (nucleation mode) < 0.1 μm
-Large aerosol (accumulation mode) 0.1 to 2 μm (most important for radiation)
-Giant aerosol (coarse mode) > 2 μm



Clouds

Types:
- Cumulonimbus: deep thunderstorms
- Cirrus: high altitude (cold) ice clouds
- altostratus, altocumulus: midlevel
- stratus and cumulus: low (boundary layer) (usually) liquid clouds.

Cloud Microphysics

- Water droplets
- radius: 2 - 40 μm radius (typical:10 μm)
- Concentration: 50 -500 cm^{-3}
- Liquid water content: 0.03 -3.0 g m^{-3}

- Ice clouds -non-spherical (hexagonal) particles
- Size: length 10 -1000 μm (typical 100 μm)
- Concentration: 1 -1000 $liter^{-1}$
- Ice water content: 0.001 -0.1 g m^{-3}

Particle Size Distributions

- Size distribution $n(r)$ is the number of particles per volume per radius interval.
- Typical units: cm^{-3} μm^{-1}
- Total number of particles per volume: $N = \int n(r)dr$
- Mass per volume of air or liquid water content (LWC) is: $LWC = \frac{4\pi}{3} \rho \int r^3 n(r)dr$

Scattering from a Distribution of Particles

- Integrate over particle size distributions and multiple particle types.

Extinction (or optical depth):

$$\beta_{ext} = \int_0^\infty C_{ext} n(r)dr = \int_0^\infty \pi r^2 Q_{ext} n(r)dr$$

Single scattering albedo: $\varpi_o = \beta_{sca} / \beta_{ext}$