Problem

Determine the flux divergence at every point in an infinite, absorbing medium according to the two-stream theory. The medium is illuminated from above. As a way of checking your result integrate the flux divergence over the entire medium. You should obtain an expression you could have written down immediately without doing any integration.

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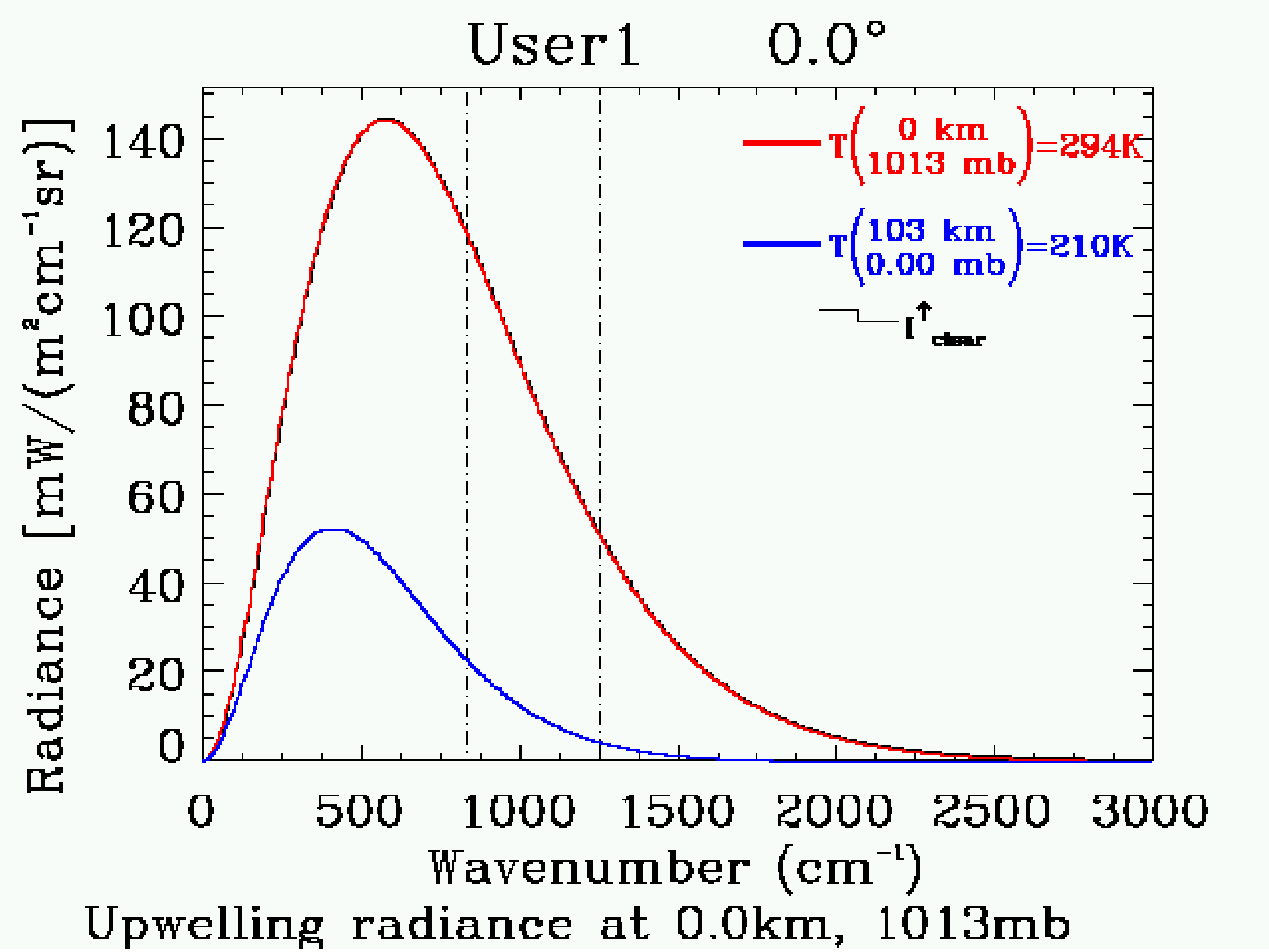
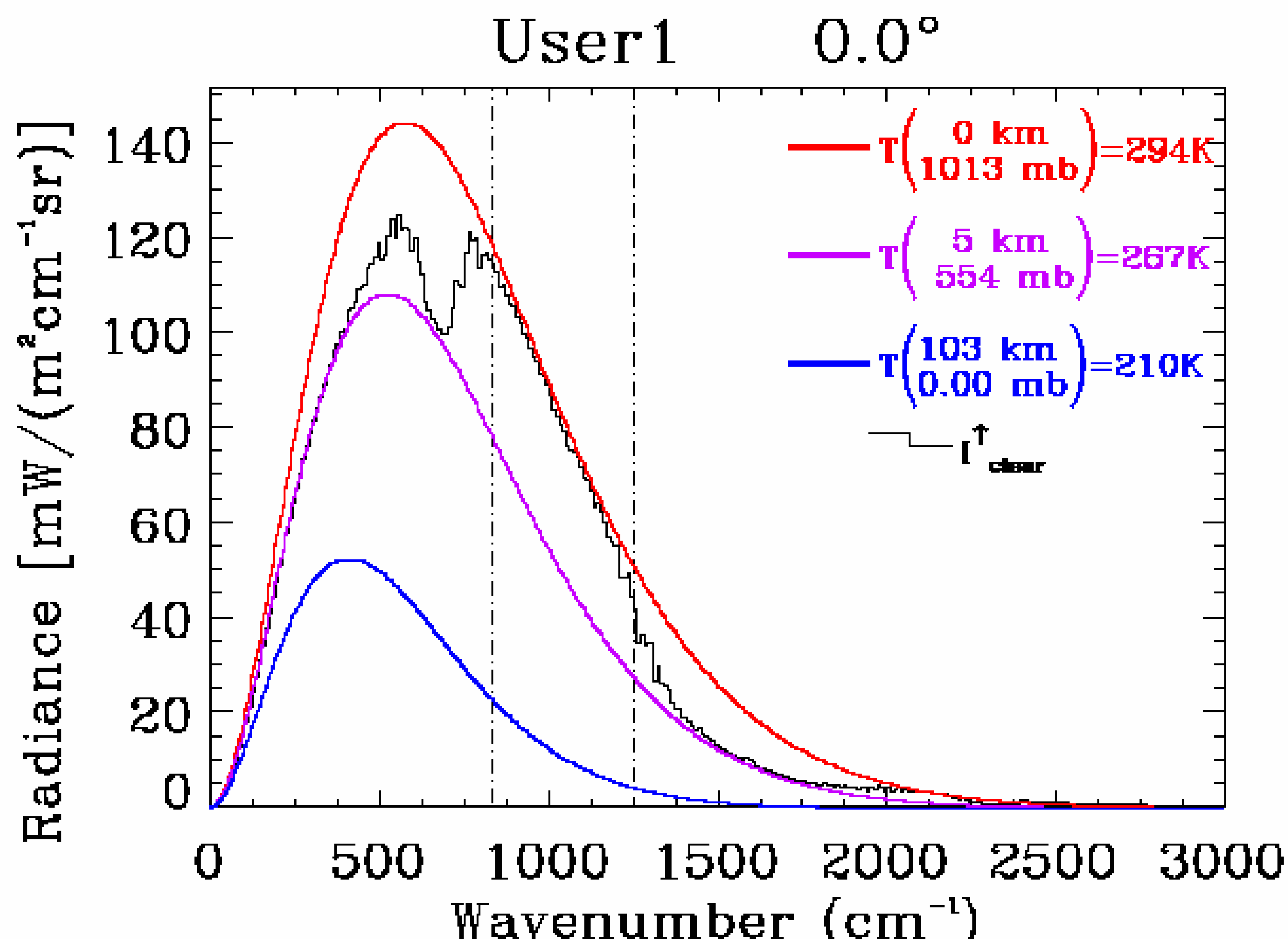
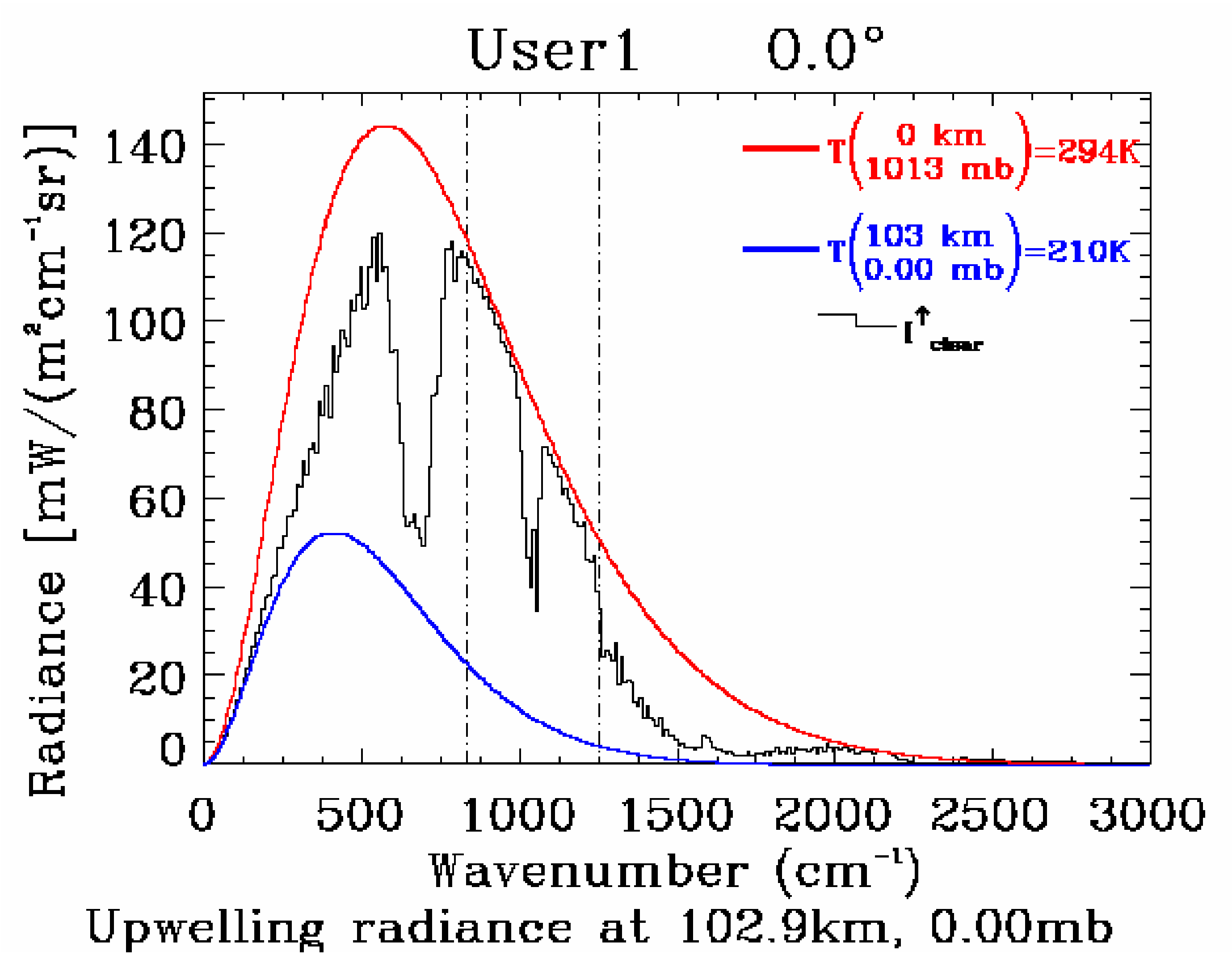


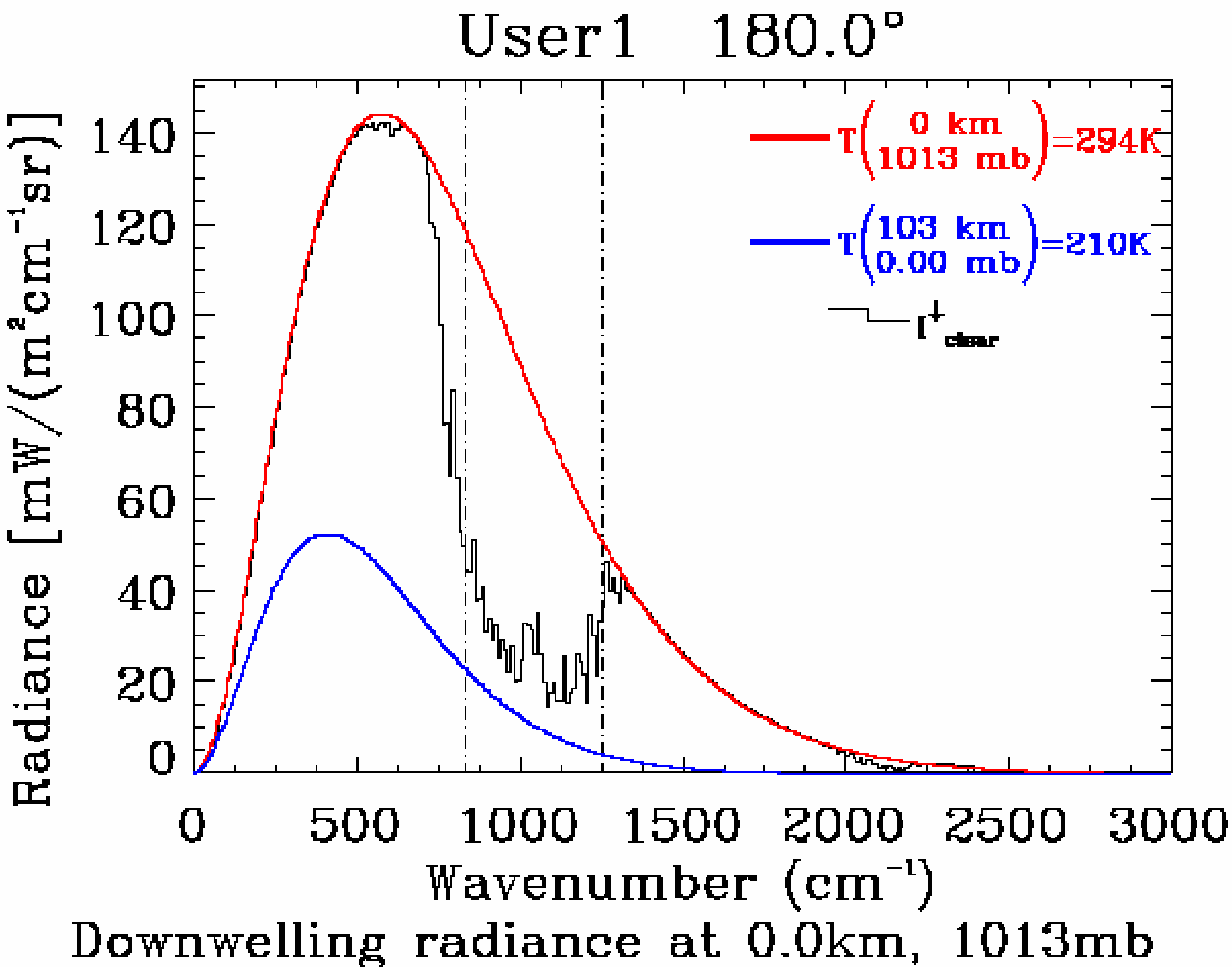
Plot the upwelling radiance spectrum (at 0 º) for the mid-latitude summer atmosphere. Explain the differences between the upwelling radiance spectra at 0 km, 5 km, and the top of the atmosphere.



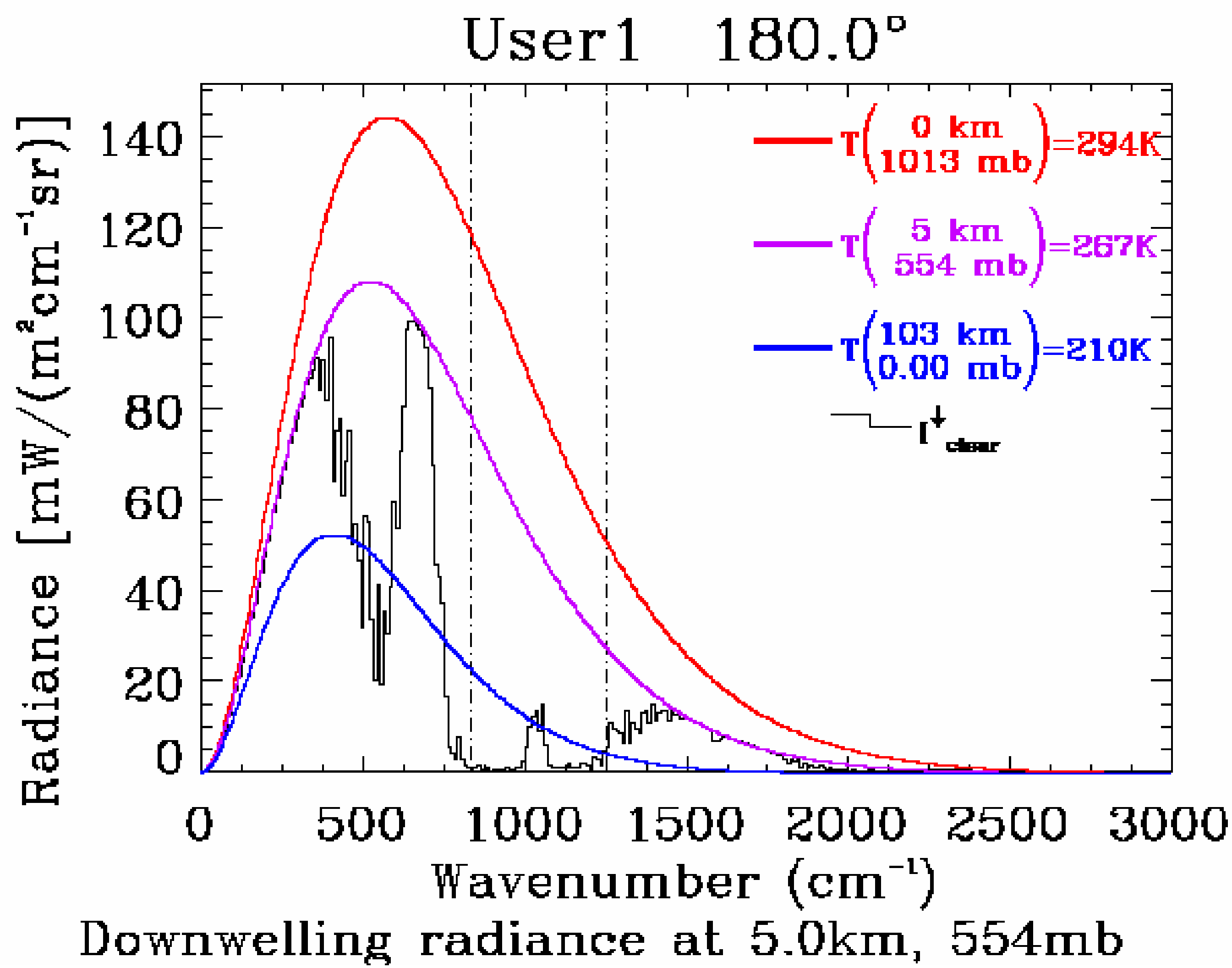
Upwelling radiance at 5.0km, 554mb

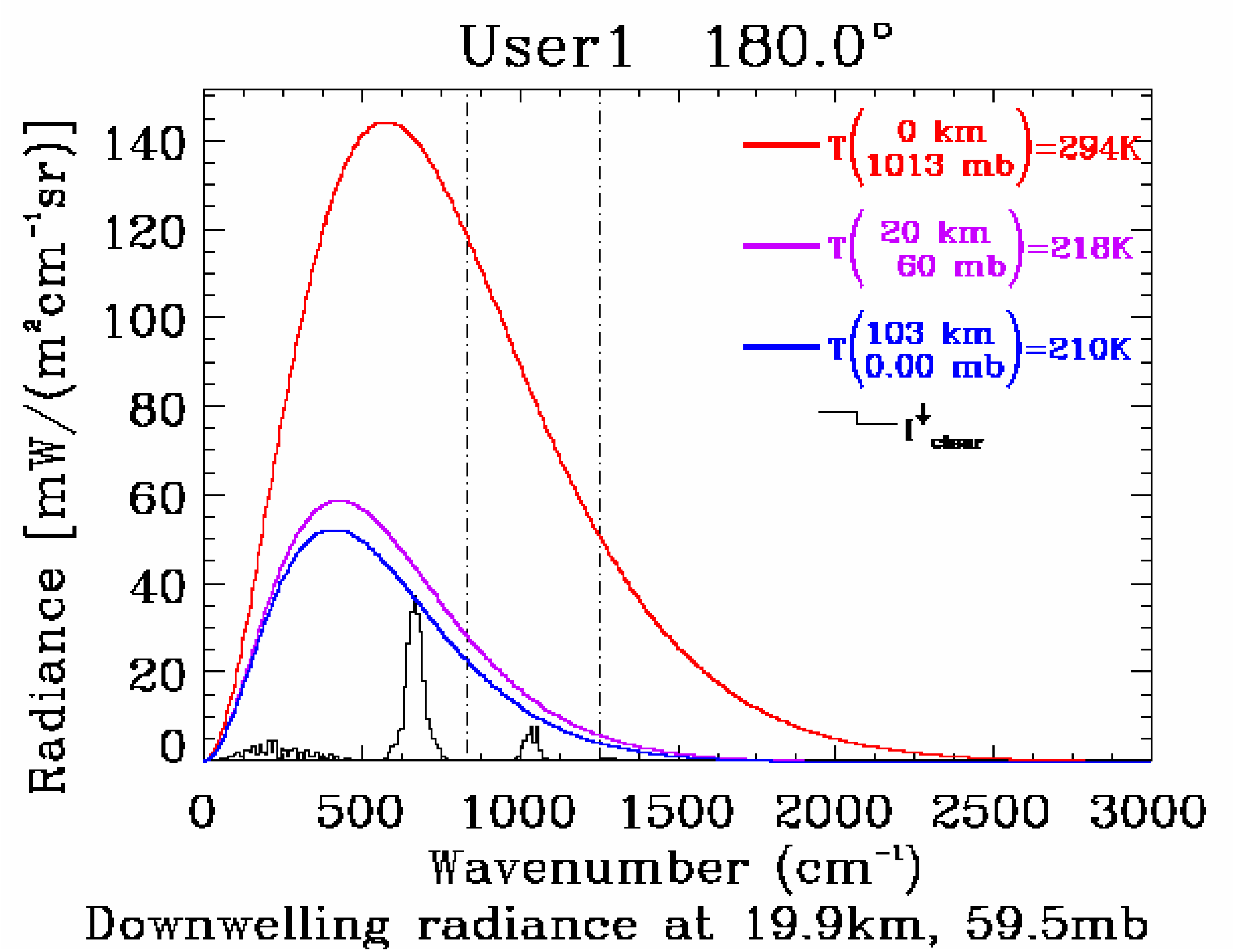

User1 0.0°
T(0 km 1013 mb)=294K
T(103 km 0.00 mb)=210K
I↑clear
Radiance [mW/(m²cm⁻¹sr)]
140
120
100
80
60
40
20
0
0
500
1000
1500
2000
2500
3000
Wavenumber (cm⁻¹)
Upwelling radiance at 102.9km, 0.00mb

- The upwelling radiance at the surface is Planck function at 294 K , the surface T (surface is a blackbody).
- The upwelling radiance at 5 km:
  - the upwelling radiance matches 294 K Planck function in the 800 to 1200 cm^{-1} window (little absorption/high transmission).
  - Below 400 cm^{-1} (H_2O rotational band), 600-700 cm^{-1} (CO_2 band), and 1500-1800 cm^{-1} (H_2O vibrational band) radiance matches 267 K Planck function (temperature at 5 km): emission originates from close to the level.
  - In the adjacent regions radiance is between the two Planck functions because some of the contributing radiation originates at lower altitudes or from the surface.
- The upwelling radiance at top-of-atmosphere:
  - substantial depressions from the surface radiance below 600 cm^{-1} from the H_2O rotational band, from the CO_2 band around 667 cm^{-1} , from the ozone band around 1000 cm^{-1}, and from 1200 to 2000 cm^{-1} from the water vapor vibrational band.
  - The radiance decrease is greater for the CO_2 and O_3 bands because their weighting functions peak in the very cold stratosphere.
  - The radiance decrease in the water vapor bands is less because the weighting functions peak in the middle to upper troposphere because nearly all the water vapor is in the low to mid troposphere.



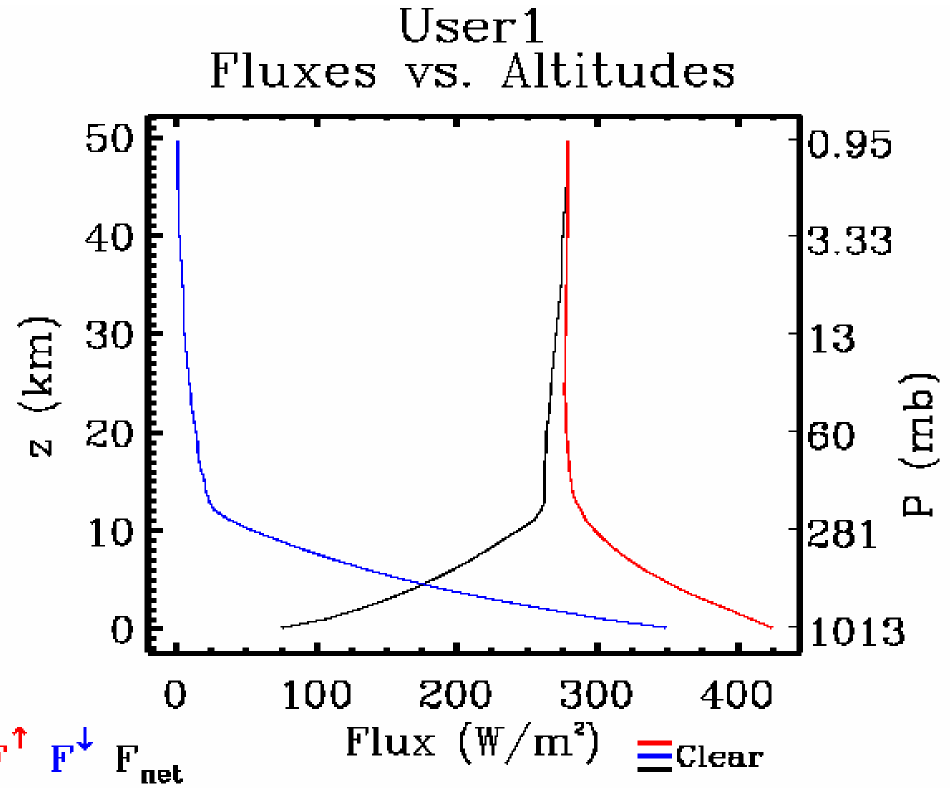
Plot the downwelling radiance spectrum for the midlatitude summer atmosphere. Explain the differences across the spectrum between the downwelling radiances at 20, 5, and 0 km.

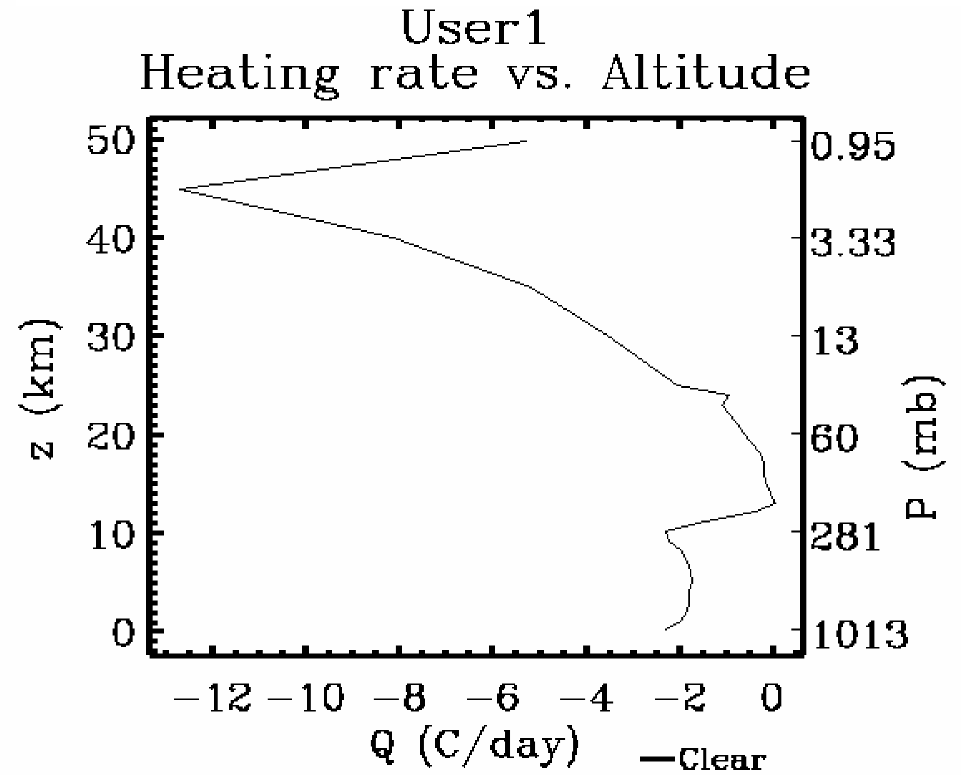

User1 180.0°
Radiance [mW/(m²cm⁻¹sr)]
140
120
100
80
60
40
20
0
0
500
1000
1500
2000
2500
3000
Wavenumber (cm⁻¹)
T(0 km, 1013 mb)=294K
T(5 km, 554 mb)=267K
T(103 km, 0.00 mb)=210K
Downwelling radiance at 5.0km, 554mb



User1 180.0°
Radiance [mW/(m²cm⁻¹sr)]
140
120
100
80
60
40
20
0
0
500
1000
1500
2000
2500
3000
Wavenumber (cm⁻¹)
T(0 km, 1013 mb)=294K
T(20 km, 60 mb)=218K
T(103 km, 0.00 mb)=210K
I↓clear
Downwelling radiance at 19.9km, 59.5mb

- The downwelling radiance at at 0 km:
  - Matches the 294 K Planck curve except in the 8 to 12 μm window.
  - Absorption, mainly due to water vapor near the surface, is so strong that the transmission falls off very rapidly with height; emission originates in the warm atmosphere near the surface.
  - Even in the 8 to 12 μm window there is substantial downwelling radiance due to water vapor continuum.
- The downwelling radiance at 5 km:
  - radiance matches the 267 K Planck function below 400 cm^{-1} (water vapor), from 600 to 700 cm^{-1} (CO_2), and from 1500 to 1900 cm^{-1} (water vapor).
  - strong absorption causes emission to originate close to 5 km.
  - The radiance in ozone band is low because ozone emission is from colder stratosphere at cold and because the absorptivity of the ozone band is less than 100%.
  - The rest of the 8-12 μm window radiance is near zero because there is not much water vapor above 5 km.
- The downwelling radiance at 20 km:
  - Near zero except for 15 μm CO_2 band and the 9.6 μm ozone band
  - Radiance is even low in these bands because of low temperatures an emissivity is less than one (low absorber abundance above 20 km).





Plot the spectrally integrated fluxes and heating rate for the midlatitude summer atmosphere. Plot as a function of altitude and use 49 km for the upper level so only the troposphere and stratosphere are plotted. For a different perspective, change the y-axis of theplots to pressure.

- Why does the downwelling irradiance have a change in slope around 200 mb?
  - The downwelling irradiance increases much more quickly below about 12 km due to water vapor emission, since the water vapor concentration is much greater in the troposphere.
  - Water vapor absorption occurs in a much larger fraction of the spectrum than does CO_2 and O_3, which are important in the stratosphere.
- Why is the net flux slope and heating rate fairly constant from 300 to 1000 mb even though the water vapor density increases by two orders of magnitude?
  - The slope of net flux and the cooling rate are nearly constant below 12 km because there is a very wide range in the strength of water vapor absorption across the spectrum (especially in the pure rotational band).
  - At any particular altitude, (and water vapor amount above it), there is a significant part of the spectrum for which the cooling to space term is large (moderate transmission). This is apparent in the spectral heating rate profile contour in question 4.

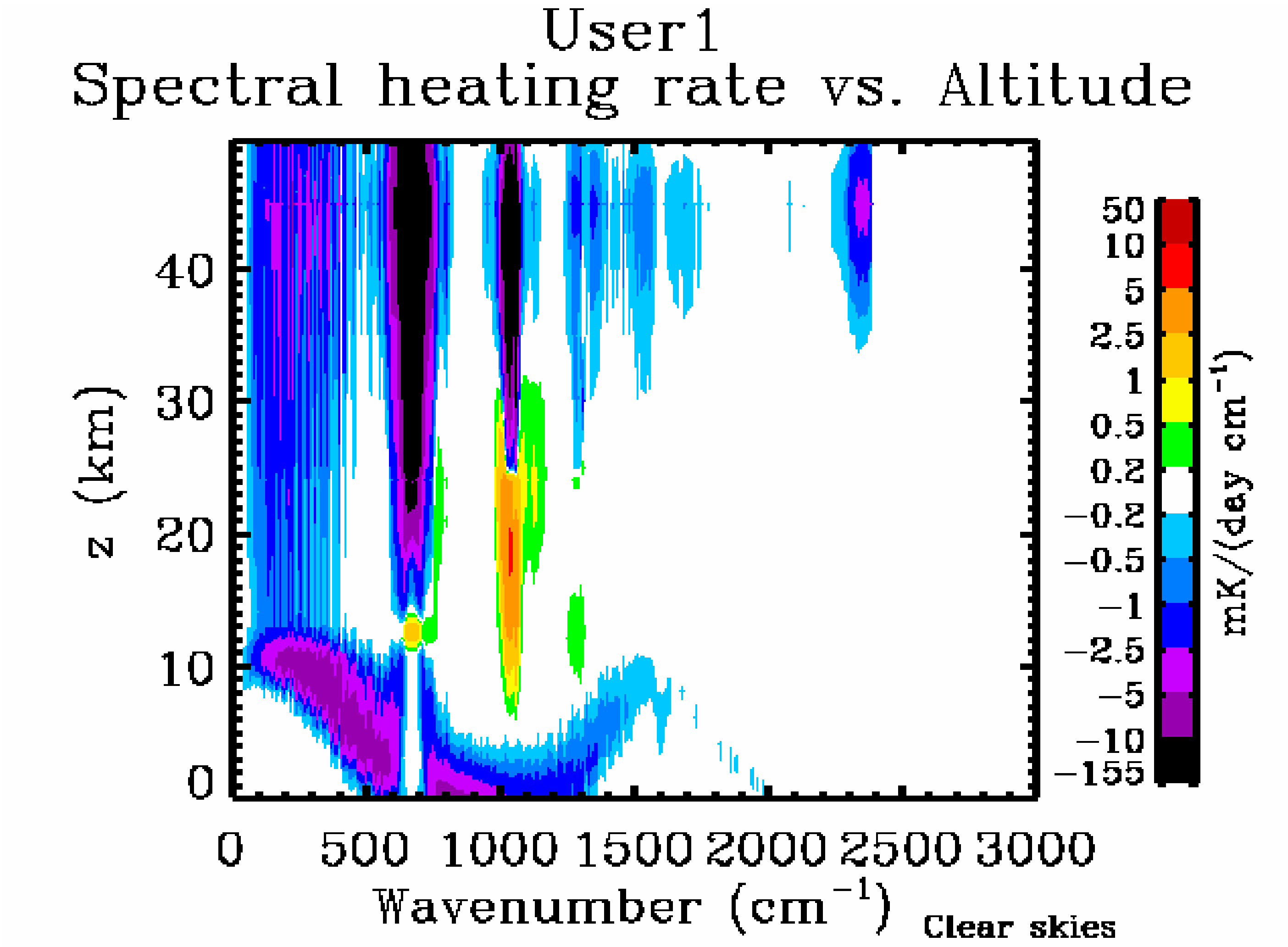
Calculate the integrated net flux divergence (ΔF_{net} in $W\ m^2$) in the troposphere and above. Note that the total atmospheric divergence does not equal the flux emitted at the top of the atmosphere – what else is cooling, i.e. what makes the longwave flux balance?

P (mb)	Alt. (km)	F_{up} ($W\ m^{-2}$)	F_{down} ($W\ m^{-2}$)	F_{net} ($W\ m^{-2}$)	ΔF_{net} ($W\ m^{-2}$)
1013	0	423.5	348.2	75.3	
					186.5
179	13	284.8	23.0	261.8	
					17.1
0	103	278.9	0.00	278.9	

The flux divergence in the stratosphere is relatively small because of the limited spectral range of the absorbers there (CO_2 and O_3). The total atmospheric flux divergence is 203.6 $W\ m^{-2}$, which is less than the outgoing top of atmosphere flux of 278.9 $W\ m^{-2}$. This is because the surface is also cooling, and loses a net flux of 75.3 $W\ m^{-2}$, which makes the longwave radiation budget balance.

For what layer does the cooling rate peak? Explain, considering the results in c).

- The cooling rate peaks in the upper stratosphere layer from 45 to 50 km due to emission by CO_2 and O_3 from the warm stratopause region (> 270K). The net flux divergence from this layer is small, but the density is about 1/1000 of the surface density, so the cooling rate is large.

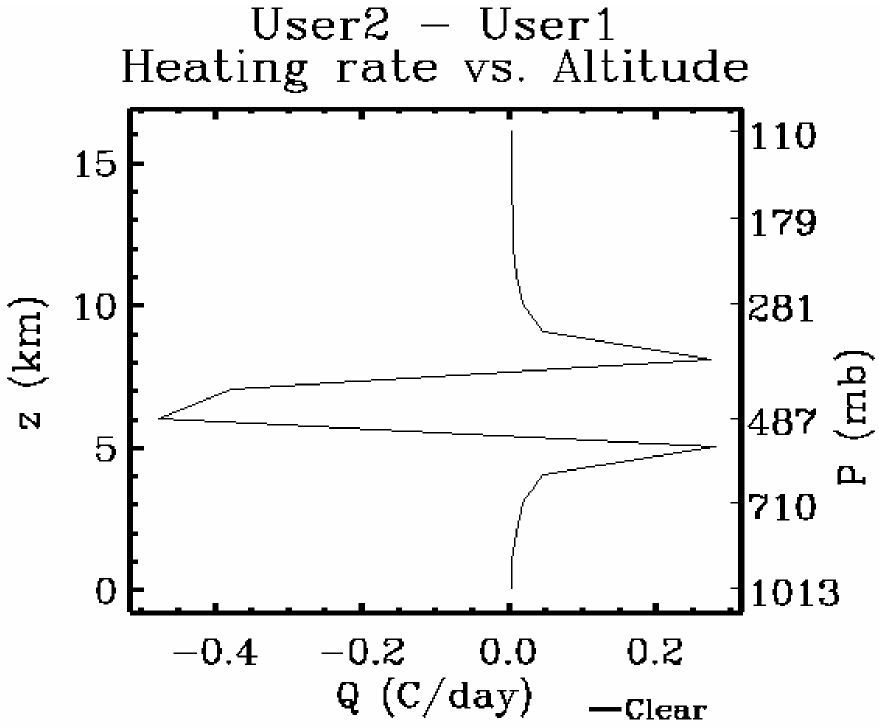


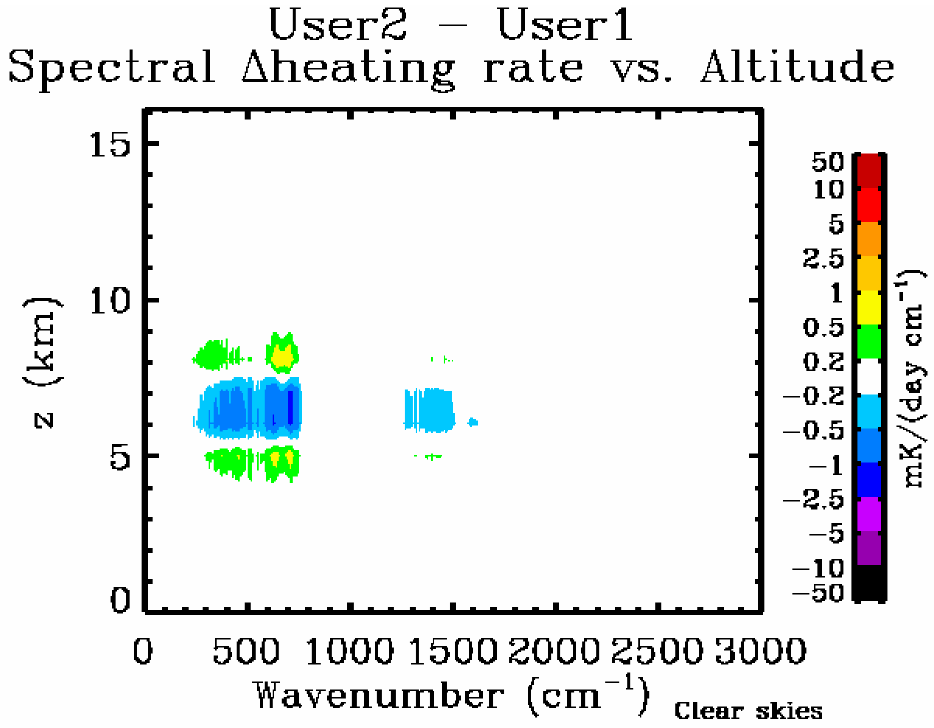
- *Look at the heating rate in more detail for midlatitude summer by making a Clough spectral heating rate profile plot (use an upper altitude of 49 km).*

- The major cooling region in the troposphere from 100 to 900 cm^{-1} is due to water vapor.
- Because water vapor absorption line strength decreases with wavenumber in the pure rotational band it takes more water vapor to reach transmission values around 50%: the peak of the weighting function and thus the peak in the cooling to space term drops from the upper troposphere to near the surface.
- The altitude of water vapor emission increases around the 1600 cm^{-1} vibrational band center, but T (and thus the Planck emission) is low so the cooling is less intense.
- There is also cooling due to water vapor in the upper stratosphere (below 500 cm^{-1} and around 1600 cm^{-1}) due to emission from the centers of very strong lines at the warm stratopause temperature.
- There are very high cooling rates in the upper stratosphere around 700 cm^{-1} from CO_2 and
- around 1000 cm^{-1} from ozone.
- The cooling rate is less in the lower stratosphere because the transmission to space (cooling to space term) is lower. Also the stratopause region is warm, so there is more emission to space.
- There is significant cooling due to 4.3 μm CO_2 band (2400 cm^{-1}).
- The major region of heating is in the lower stratosphere around 1000 cm^{-1}. This is due flux exchange between the warm surface and the cold lower stratosphere in the ozone band. There is little ozone in the troposphere so there is high transmission between the lower stratosphere ozone and the boundary layer water vapor and surface. The upwelling flux from the warm surface is larger than the downwelling flux from the cold stratosphere, so the flux exchange heats the ozone.
- There is a small region of heating in the 15 μm CO_2 band at 13 km, which is due to flux exchange with adjacent warmer layers.

Look at the effect of single level temperature and water vapor perturbations on the heating rate profile. Use an unmodified midlatitude summer for profile 1. For profile 2, modify the temperature by adding 5 K to the 7 km level. For profile 3, modify the water vapor by doubling the value at 7 km.

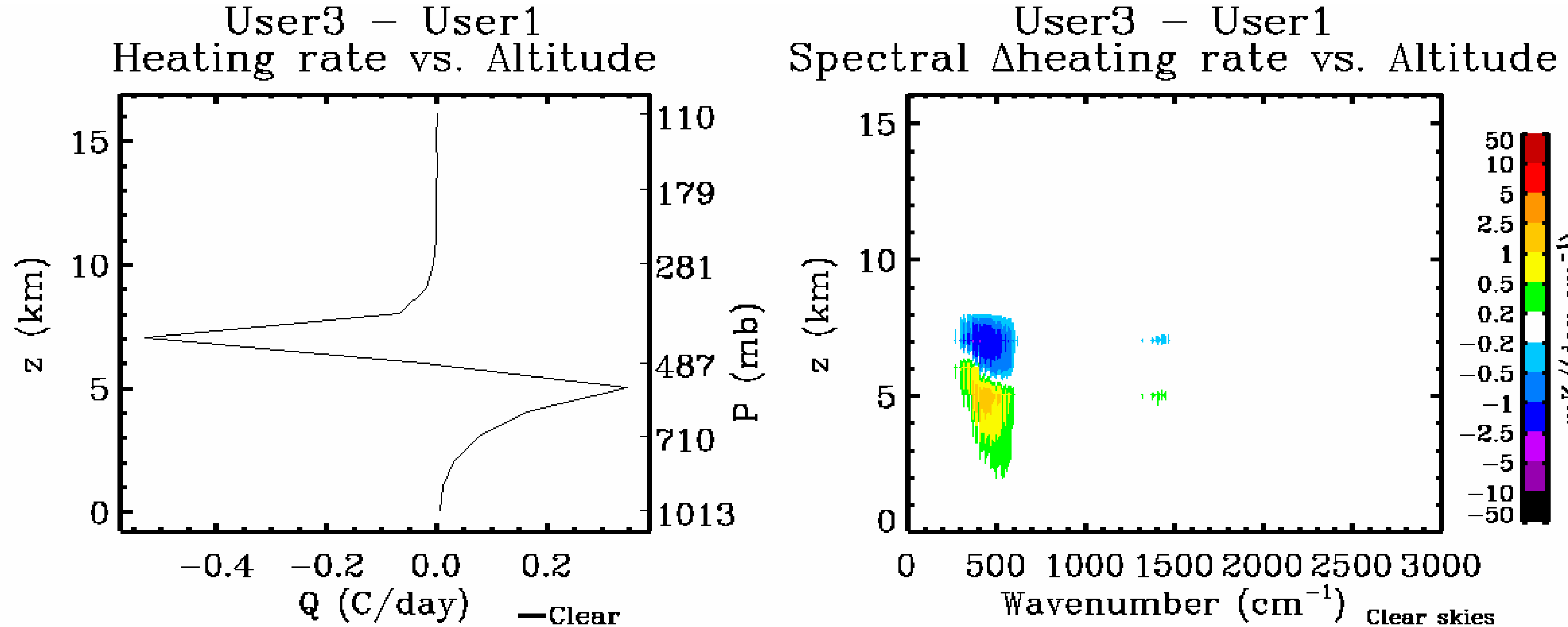
- Calculate the change in downwelling irradiance at the surface and outgoing at top of atmosphere for the temperature and water vapor perturbations.
  - At the surface:
    - Temperature: $\Delta F\downarrow(0) = 348.24 - 348.21 = 0.03 \text{W m}^{-2}$
    - Water vapor: $\Delta F\downarrow(0) = 348.27 - 348.21 = 0.06 \text{ W m}^{-2}$
  - At the top of atmosphere:
    - Temperature: $\Delta F\uparrow(\infty) = 279.80 - 278.87 = 0.93 \text{W m}^{-2}$
    - Water vapor: $\Delta F\uparrow(\infty) = 277.58 - 278.87 = -1.29 \text{ W m}^{-2}$
- Briefly explain the causes of the TOA flux changes. Why does the downwelling flux at the surface barely change?
  - T perturbation increases TOA flux due to the larger Planck emission at 7 km.
  - The water vapor perturbation decreases the TOA irradiance because the transmission from the surface and lower atmosphere is decreased by the larger water vapor absorption. There is more emission from the colder 7 km layer.
  - Downwelling irradiance at surface does not change because nearly all of the increased emission at 7 km is absorbed before it reaches the surface.
  - There is increased emission from the H_2O and CO_2 bands, but radiation in these bands is heavily absorbed by the large absorber amounts in the lower atmosphere.





Plot the broadband heating rate profile of the difference between the temperature perturbed and the original sounding. Explain the change in the longwave heating rate profile using the flux exchange concept. It may be helpful to make a Clough spectral heating rate profile plot of the difference.

- The longwave cooling rate change shows an increase in the cooling rate in the layer and a decrease in adjacent layers. The layer cooling rate increases because it emits more at the higher temperature, while it absorbs the same amount from other layers. The nearby layers absorb some of the additional emission and therefore, have less cooling than before.
- This is the flux exchange between layers concept. There has to be a temperature difference between 4 layers, significant absorptivity/emissivity of the layers, and transmission between the layers.
- The transmissivity requirement often limits the flux exchange to nearby layers.



Plot the broadband heating rate profile of the difference between the water vapor perturbed and the original sounding Explain the change in the longwave heating rate profile.

- The longwave cooling rate is increased in the layer above 7 km as the emissivity is increased by the extra water vapor. The emitted irradiance is readily transmitted to space due to the dry atmosphere above.
- The cooling rate is decreased in the layer below 7 km as more upwelling irradiance from below is absorbed.
- The extra emitted downwelling irradiance is absorbed in the layers below the perturbation, decreasing the cooling rate there.

Look at the effect of doubling carbon dioxide. Use midlatitude summer for the first profile. Double the CO_2 concentration in the second profile.

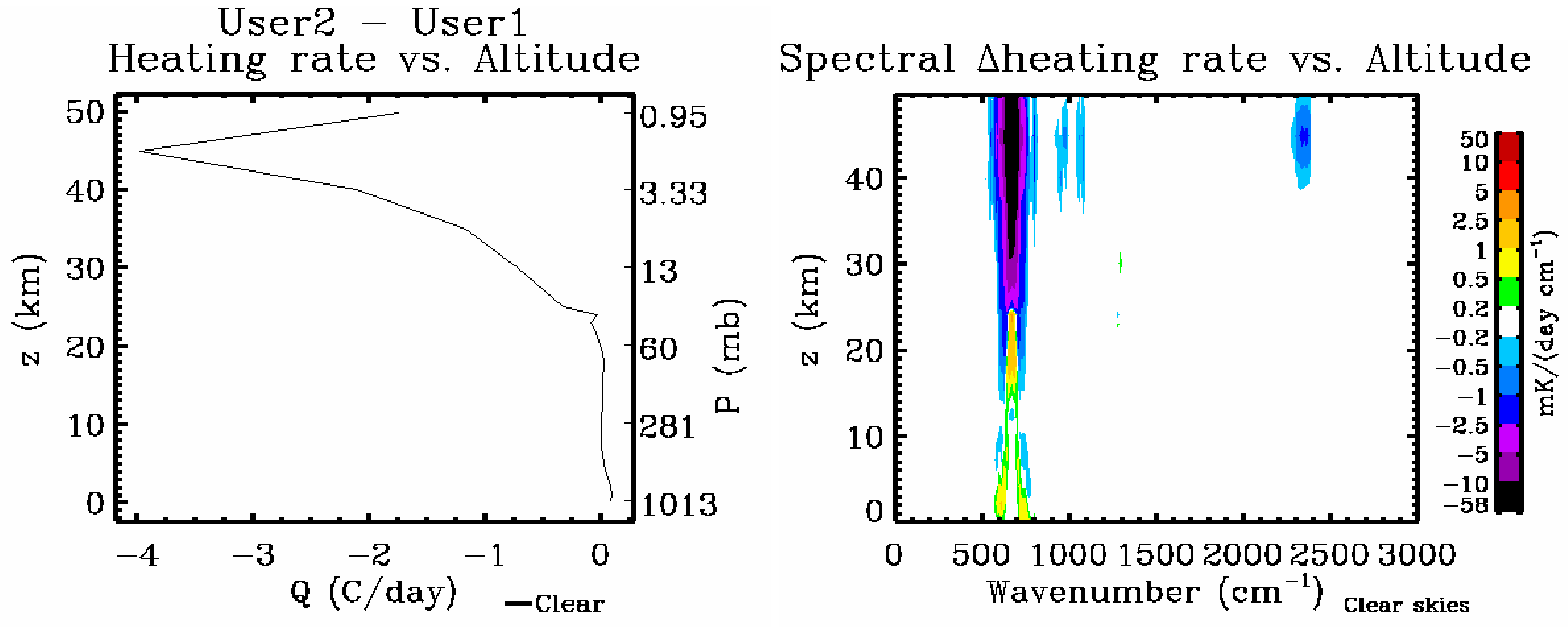
- *Calculate the change in downwelling irradiance at the surface and outgoing at the top of atmosphere from doubling CO_2. Briefly explain the outgoing flux changes.*

  – Change in F↓ at the surface is 350.0 − 348.2 = 1.8 W m^{-2}.

  – Change F↑ at the top of atmosphere is 276.13 − 278.87 = −2.74W m^{-2}.

  – The increase in CO_2 absorption raises the altitude of outgoing emission to where the temperatue is lower, decreasing the upwelling TOA irradiance.

  – The downwelling irradiance at surface increases because the increased CO_2 absorption lowers the emission level to warmer temperatures.



Plot the broadband heating rate profile of the difference between the doubled CO2 and original sounding. Explain the change in the longwave heating rate profile.

- The most noticeable change is the increase in cooling rate in the upper stratosphere due to increased emission to space by CO_2.
- The largest temperature change in the entire atmosphere from doubling carbon dioxide is a decrease in the upper stratosphere.

N-Stream Theory of Radiative Transfer

- Recall two-stream equations:

$$\frac{dF_{\downarrow}}{dz} = -(\kappa + \beta)F_{\downarrow} + \beta(p_{\downarrow\downarrow}F_{\downarrow} + p_{\uparrow\downarrow}F_{\uparrow}) \qquad \frac{dF_{\uparrow}}{dz} = (\kappa + \beta)F_{\uparrow} - \beta(p_{\uparrow\uparrow}F_{\uparrow} + p_{\downarrow\uparrow}F_{\downarrow})$$

Write in compact form:

$$\mu_k \frac{dF_k}{dz} = -(\kappa + \beta)F_k + \beta\sum p_{jk}F_j, k = 1, 2$$

- p_{jk} is probability that a photon in the direction j is scattered in the direction k.
- j and k take on only two values: 1 corresponds to downward, 2 corresponds to upward
- μ_k is the cosine of the angle between the positive z-axis and either of the two allowed directions.
- μ_k = 1 or -1.

N-Stream Theory of Radiative Transfer

- Generalize to *N* streams:

$$\mu_k \frac{dF_k}{dz} = -(\kappa + \beta)F_k + \beta \sum q_{jk} F_j, \, k = 1, 2, ..., N \text{ where } q_{jk} = \pm p_{jk} \frac{\mu_k}{\mu_j}$$

  - p_{jk} is the probability that, given that a photon in direction j is scattered, it is scattered in direction k
  - the μ_k are direction cosines.
  - The symbol ± indicates that the sign is chosen to make the entire quantity positive
- Radiation field is approximated as a set of mono-directional beams
- Another approximation: probability of scattering exactly in one direction is zero because scattering directions vary continuously.
- The correct probability is a distribution function giving the probability of scattering into a finite, although possibly small, set of directions (i.e., small solid angle).
- Note also the singularity at $\pi/2$ ($\mu=0$). In the derivation we assume that $\beta\Delta z/\mu \ll 1$. When $\mu = 0$, this condition cannot be satisfied.

Equation of Transfer

- Must go beyond N-stream theory
- Taking the limit of the sum on N-stream as the number of terms in goes to infinity leads to the integral form the equation of transfer.
- Directional derivative (rate of change of *f* along some direction in three-dimensional space):

$$\frac{df}{ds} = \lim_{s \to 0} \frac{f(\mathbf{x} + s\Omega) - f(\mathbf{x})}{s}$$

Ω is a unit vector specifying direction

$$\frac{df}{ds} = \mathbf{\Omega} \cdot \nabla f$$

- rate of change of radiance *L* along a direction specified by unit vector Ω is the corresponding directional derivative:

$$\frac{dL}{ds} = \mathbf{\Omega} \cdot \nabla L(\Omega)$$

(we will derive equation of transfer for radiance rather than radiance because it is function of direction)

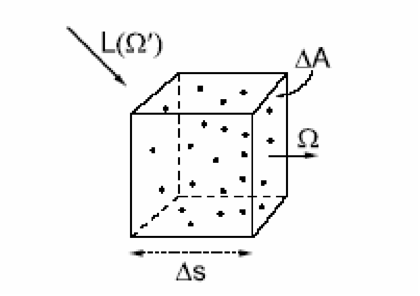
Equation of Transfer

- Attenuation of *L* over a distance Δs:

$$\Delta L(\Omega) = -L(\Omega)(\kappa + \beta)\Delta s$$

- Emission: $\kappa \Delta s\, P_e / \pi$

- Scattering:



- Power scattered in all directions: $L(\Omega')\Delta\Omega' N \Delta s \Delta A C_{sca} = L(\Omega')\Delta\Omega' \beta \Delta s \Delta A$

- Scattered radiance in direction Ω: $L(\Omega')\Delta\Omega' \beta p(\Omega', \Omega)\Delta s$

- Contribution from all directions: $\beta \Delta s \int_{4\pi} L(\Omega') p(\Omega', \Omega) d\Omega'$

Equation of Transfer

- The total change of $L(\Omega)$ along Δs is $\Omega \cdot \nabla L(\Omega)\,\Delta$s, which when set equal to the sum of attenuation, emission, and scattering:

$$\mathbf{\Omega} \cdot \nabla L(\Omega) = -(\kappa + \beta) L(\Omega) + \beta \int_{4\pi} L(\Omega') p(\Omega', \Omega) d\Omega' + \kappa P_e / \pi$$

- Phase function is normalized: $\int_{4\pi} p(\Omega', \Omega) d\Omega' = 1$

- In the absence of scattering and absorption, the radiance L along any direction is invariant (i.e., the directional derivative is zero)
- For no absorption (κ = 0) and an isotropic radiation field (L independent of direction) $\Omega \cdot \nabla$ L = 0: the radiance field is the same everywhere.
- This is consistent with what we obtained in chapter 5 with the two-stream theory for a non-absorbing medium: when the radiation field was uniform it also was isotropic.

Plane-Parallel Medium

- hypothetical *continuous* medium having properties that vary in only one direction
- extends to infinity in directions in a plane, and hence is designated as *plane-parallel*.
- No such medium exists.
  - atmosphere is spherical.
  - Real clouds are finite, properties vary in all directions,
- assume that emission within media is negligible: consider external sources of radiation only
- ignore the finite time it takes radiation to propagate
  - as sunlight changes, the radiation field everywhere in a cloud or in air adjusts almost instantaneously to these changes

Solutions to the equation of transfer: Plane parallel media

- numerical plane-parallel radiative transfer methods use *discrete ordinates*
- Start with the equation of transfer Fourier transformed in azimuth.
- Replace the scattering integral by a quadrature sum:

$$\int_a^b f(x)dx \approx \sum_{j=1}^{N} w_j f(x_j)$$

  - x_j are the *discrete ordinates* and w_j are quadrature weights

- The equation of transfer becomes a ordinary differential *matrix* equation.

Solutions to the equation of transfer: Plane parallel media

Equation of transfer:

$$\mathbf{\Omega} \cdot \nabla L(\Omega) = -(\kappa + \beta) L(\Omega) + \beta \int_{4\pi} L(\Omega') p(\Omega', \Omega) d\Omega' + \kappa P_e / \pi$$

- In 1-D, scattering only:

$$\mu \frac{dL(\mu,\phi)}{d\tau} = L(\mu,\phi) - \varpi \int_0^{2\pi} \int_{-1}^{1} p(\Theta) L(\mu',\phi') d\mu' d\phi'$$

μ=cos(z)
ϕ is azimuth angle
Θ Is scattering angle;
Θ = f(μ,ϕ)

which is similar to our discrete N-stream equation:

$$\mu_k \frac{dF_k}{dz} = -(\kappa + \beta) F_k + \beta \sum q_{jk} F_j, \; k = 1, 2, \ldots, N \text{ where } q_{jk} = \pm p_{jk} \frac{\mu_k}{\mu_j}$$

- Plane-parallel radiative transfer is often solved with a Fourier series in ϕ:

$$L(\tau,\mu,\phi) = \sum_{m=0}^{N} L_m(\tau,\mu) \cos[m(\phi_0 - \phi)]$$

m = 0 term is the azimuthal average, $L_0(\tau,\mu)$.

Solutions to the equation of transfer: Plane parallel media

- addition theorem of spherical harmonics:

$$p(\mu,\phi;\mu',\phi') = \sum_{m=0}^{N}\sum_{l=m}^{N}\varpi_l a_{lm} P_l^m(\mu) P_l^m(\mu') \cos[m(\phi_0-\phi)]$$

- Legendre series coefficients ϖ_l are defined by:

$$p(\cos\Theta) = \sum_{l=0}^{N}\varpi_l P_l(\cos\Theta)$$

Legendre polynomials

- Radiative transfer equation becomes:

$$\mu\frac{dL_m(\tau,\mu)}{d\tau} = L_m(\mu,\phi) - 2\pi\varpi\sum_{l=m}^{N} a_{lm}\varpi_l P_l^m(\mu)\int_0^{2\pi}\int_{-1}^{1} P_l^m(\mu') L_m(\tau,\mu') d\mu'$$

  – Replace integral by sum: $\int_a^b f(\mu)d\mu \approx \sum_{j=1}^{N} w_j f(\mu_j)$

- Radiances at discrete angles are up and down vectors ($L+$, $L-$); matrix form of equation of transfer

Solutions to the equation of transfer: Plane parallel media

- Adding/Doubling
  - Addition theorem used to calculate Fourier transformed phase function
  - Initialization: local reflection R and transmission T matrices for initial layer $\delta\tau$ made from phase function.
  - Doubling: use doubling formula n times to get R, T , S for homogeneous layer with $\Delta\tau = 2n\delta\tau$.
  - Adding: use adding formula to combine distinct homogeneous layers and surface together
- Diffusion Approximations
  - Transport of photons driven by concentration gradients (Fick's law)

Plane-parallel RT models

- DISORT
- MDTERP
- MODTRAN (does spherical atmospheres too?): employs various 2- and N-stream scattering models; various transmittance models
- SBDART