

Exchange of Thermal (infrared) Irradiance between Layers

Derive net flux convergence from radiative transfer equation to understand sources of heating/cooling at a level.

- The spectral net flux divergence may be obtained from RTE solution by taking height derivative of net irradiance $F_{\uparrow} - F_{\downarrow}$

$$F_{\uparrow} = \pi B(T_0)t(z=0) + \int_0^z \pi B(z') \frac{dt(z-z')}{dz'} dz'$$

$$F_{\downarrow} = \int_{\infty}^z \pi B(z') \frac{dt(z'-z)}{dz'} dz'$$

$$F_{\uparrow} - F_{\downarrow} = \pi B(T_0)t(0) + \int_0^z \pi B(z') \frac{dt(z-z')}{dz'} dz' + \int_z^{\infty} \pi B(z') \frac{dt(z'-z)}{dz'} dz'$$

Differentiate, then add, subtract terms:

$$\frac{dF}{dz} = \pi B(z) \frac{dt(z, \infty)}{dz} + \pi [B(0) - B(z)] \frac{dt(0, z)}{dz} + \int_0^z \pi [B(z') - B(z)] \frac{d^2 t(z-z')}{dz dz'} dz' + \int_z^{\infty} \pi B(z') \frac{d^2 t(z'-z)}{dz dz'} dz'$$

First term is cooling to space - often a good approximation. Cooling to space is simply the weighting function times the Planck flux and depends only on the temperature at that level. Recall the thermal-rt lab exercise showing that the level where the temperature matched to top-of-atmosphere upwelling brightness temperature was approximately $\tau = 1$.

Second term is exchange of energy with surface. This is important if large temperature difference and transmission is high. The integrals are exchange between levels below and above.

Solar Heating Rates

$$\frac{dT}{dt} = -\frac{1}{\rho C_p} \frac{dF}{dz} = \frac{g}{C_p} \frac{dF}{dp}$$

In shortwave, no emission means no cooling, only heating from absorption of solar radiation. In general, solar flux solution requires scattering, but Beer's Law extinction on solar beam is a reasonable approximation for clear sky heating rates.

Spectrally integrate Beer's Law: $F_{\downarrow} = \mu_0 \int F_0 \exp(-\tau/\mu_0) d\nu$ and $\frac{dF_{\downarrow}}{dz} = \int F_0 \beta \exp(-\tau/\mu_0) d\nu$

Chapman Profile

Assume absorption k is constant with height, density ρ is exponential: $\beta(z) = k\rho_0 \exp(-z/H)$

Monochromatic flux divergence:

$$\frac{dF_{\downarrow}}{dz} = F_0 \beta \exp(-\tau/\mu_0) = F_0 k \rho_0 e^{-z/H} \exp\left(-\frac{k\rho_0 H}{\mu_0} e^{-z/H}\right)$$

Where does this reach a max? Recall weighting functions? At $\tau = 1$ or in this general case with $1/\mu_0$ in exponent, when $\tau = 1/\mu_0$.

$$\text{Recall: } \tau(z) = k\rho_0 \int_z^{\infty} \exp(-z'/H) dz' = k\rho_0 H \exp(-z/H)$$

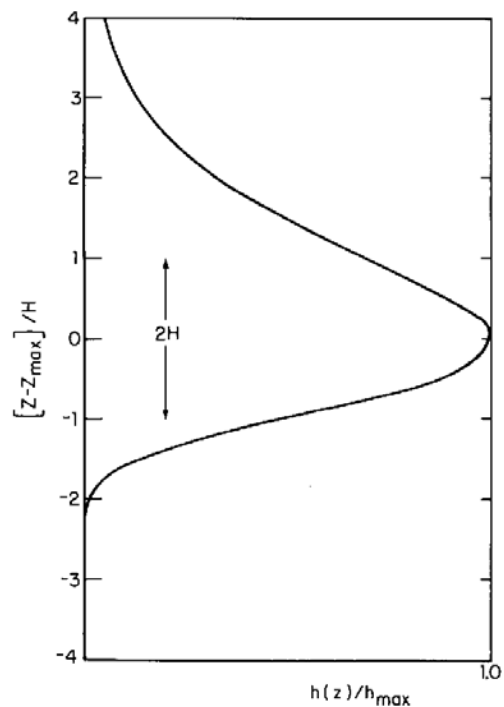
$$\left(\frac{dF_{\downarrow}}{dz}\right)_{\max} = \frac{\mu_0 F_0}{H} e^{-1} \text{ and this occurs at } z = H \ln\left(\frac{k\rho_0 H}{\mu_0}\right) \text{ and } \rho = \frac{\mu_0}{kH}$$

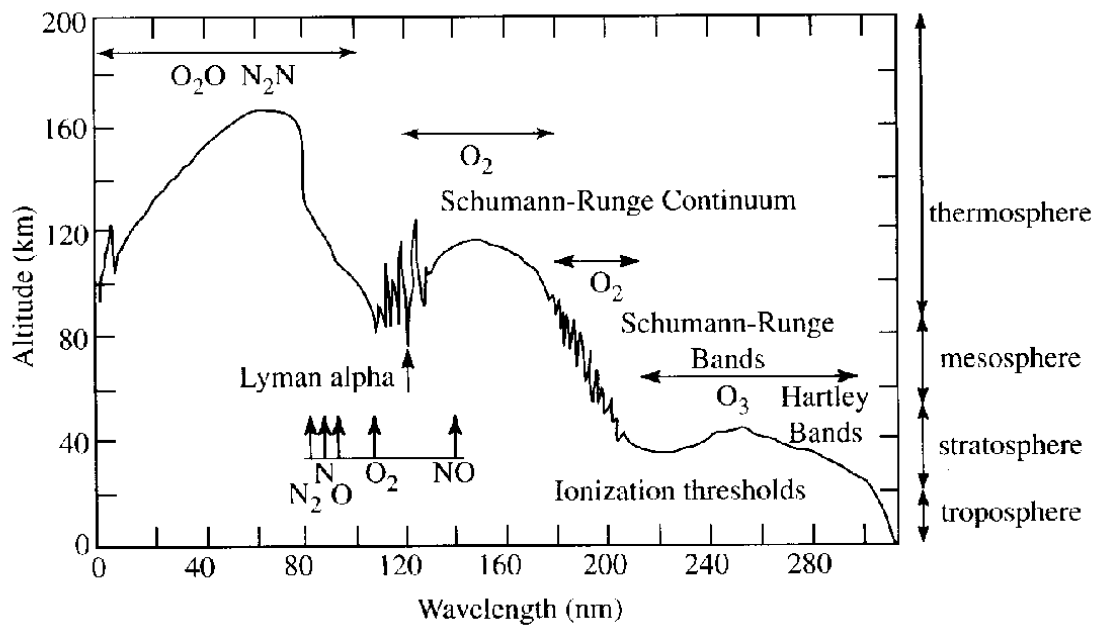
This is relevant for well mixed gases, e.g. O_2 in the far ultraviolet.

Higher absorption coefficient: max divergence occurs at higher z and lower ρ .

Most monochromatic flux convergence occurs in a layer about 2 scale heights H thick. For a range of absorption coefficients (range of wavelengths) the layer is wider.

A Chapman layer where the solar energy is absorbed. The altitude is scaled by the absorber scale height H and the net flux convergence $h(z)$ is normalized to the maximum value. [Goody and Yung, Fig. 6.9]





Atmospheric penetration depth versus wavelength. Horizontal arrows indicate the molecule (and band) responsible for absorption in that spectral region. Vertical arrows indicate the ionization threshold of the various species. [Thomas and Stamnes, Fig. 9.3]