Weighting functions

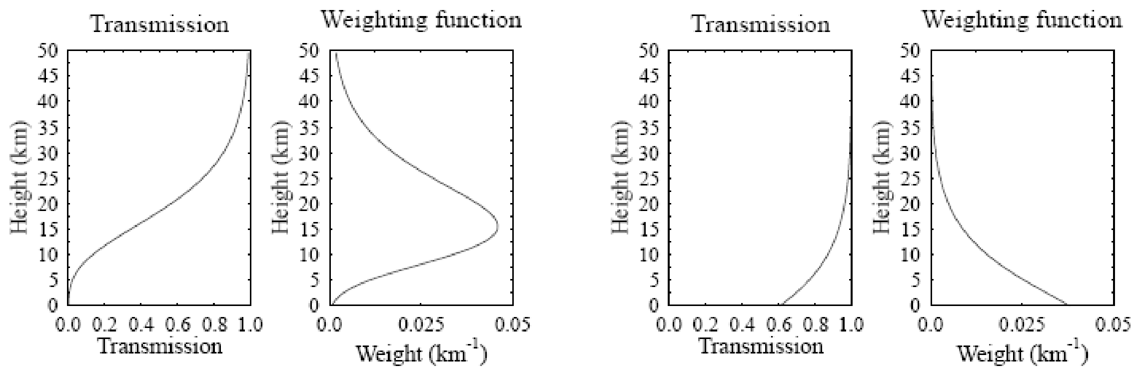
Recall solution of transfer equation, emission only:

$$L_{\uparrow}(\tau=0) = B(T_0)\exp(-\tau_0/\mu) + \int_0^{\tau_0} \exp(-\tau'/\mu) B\{T(\tau)\} d\tau'/\mu$$ where τ_0 is the total optical depth and T_0 is temperature at the surface.

Recall that $d\tau = \beta dz$ we can write: $L_{\uparrow}(\tau=0) = B(T_0)t_0 + \int_0^z B(T)\frac{dt}{dz'}dz'$ where transmittance t=exp(-τ/μ).

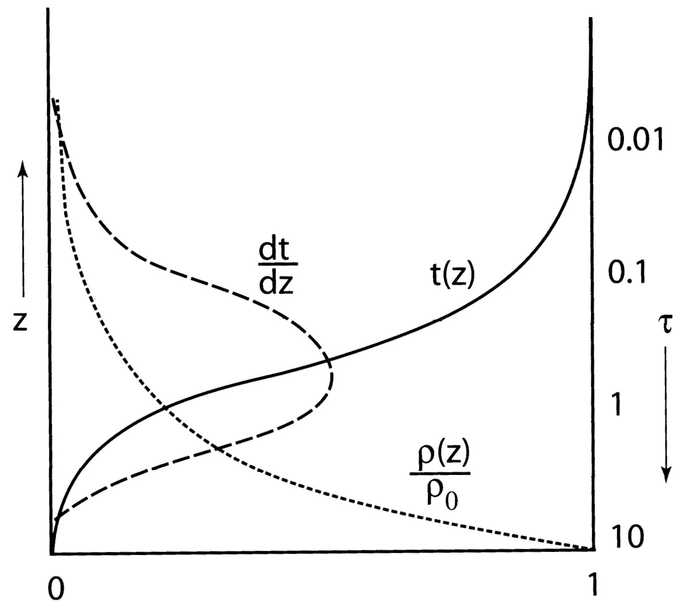
We define the derivative of transmittance with respect to a vertical coordinate (in this base, height, z) the ***weighting function***: $W(z) = \frac{dt}{dz}$

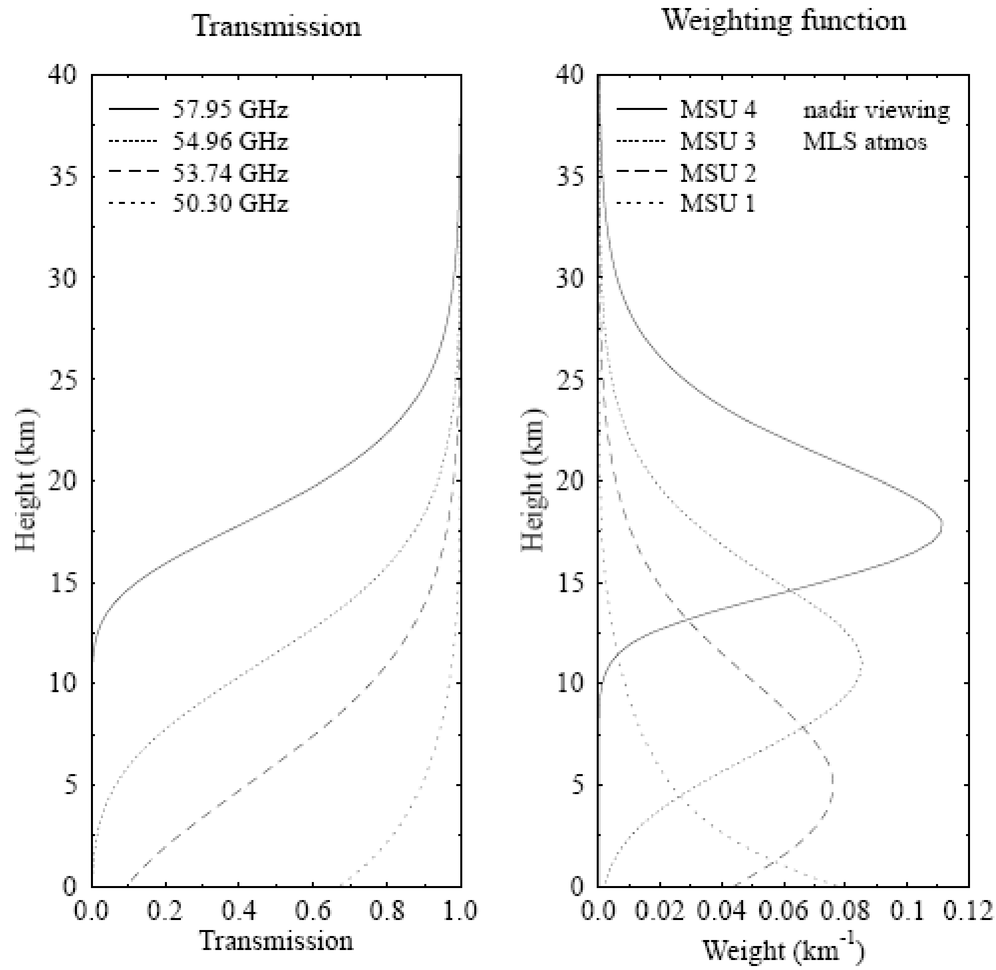
Weighting functions give the contribution to outgoing radiance from each level.



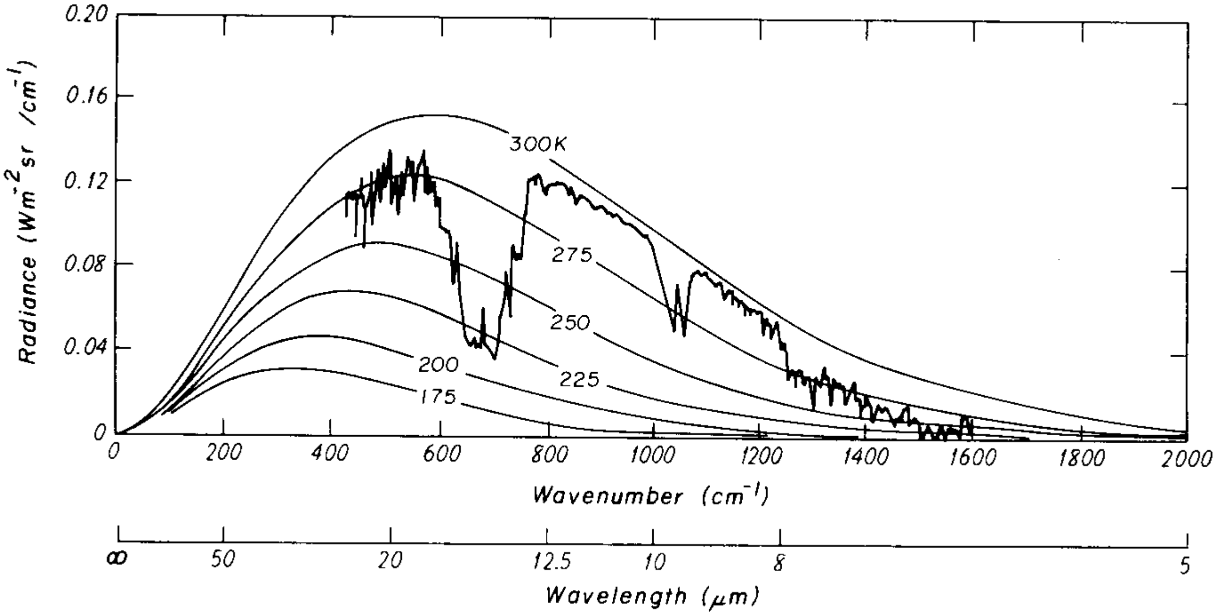
$W(z) = \frac{dt}{dz} = -t\frac{d\tau}{dz} = \beta t$ ($d\tau$ = -βdz and ignoring μ) so $W(z)$ is product of two variables, one that increases (β) as β increases and one that decreases (t) as β increases. This explains the behavior of weighting functions and where it reaches a maximum.

Where does maximum occur? Assume absorption coefficient falls off exponentially with scale height H: $\beta(z) = \beta_0 \exp-(z/H)$. Then $\tau(z) = \beta_0 \int_z^{\infty} \exp-(z'/H)dz' = \beta_0 H \exp-(z/H)$ and $W(z) = \beta_0 \exp(-z/H)t(z) = \beta_0 e^{-z/H}\exp(-\beta_0 H e^{-z/H})$. To find maximum, find where derivative with respect to z goes to 0: $\frac{d}{dz}W(z) = \frac{\beta_0}{H}e^{-z/H}\exp\left(-\beta_0 H e^{-z/H}\right)\left[\beta_0 H e^{-z/H} - 1\right] = 0$ or $\beta_0 H e^{-z/H} = 1$ so $\tau(z) = 1$. Maximum in weighting function occurs at optical depth of unity.





Transmission profiles and weighting functions referenced to space for the Microwave Sounding Unit on the NOAA polar orbiting satellites.



An upwelling Earth radiance spectrum measured by the Infrared Interferometer Spectrometer aboard the Nimbus 4 satellite. Planck radiance curves are also shown. [Liou, 1992; Fig 2.1.]

Weighting function are used in the retrieval of temperature soundings:

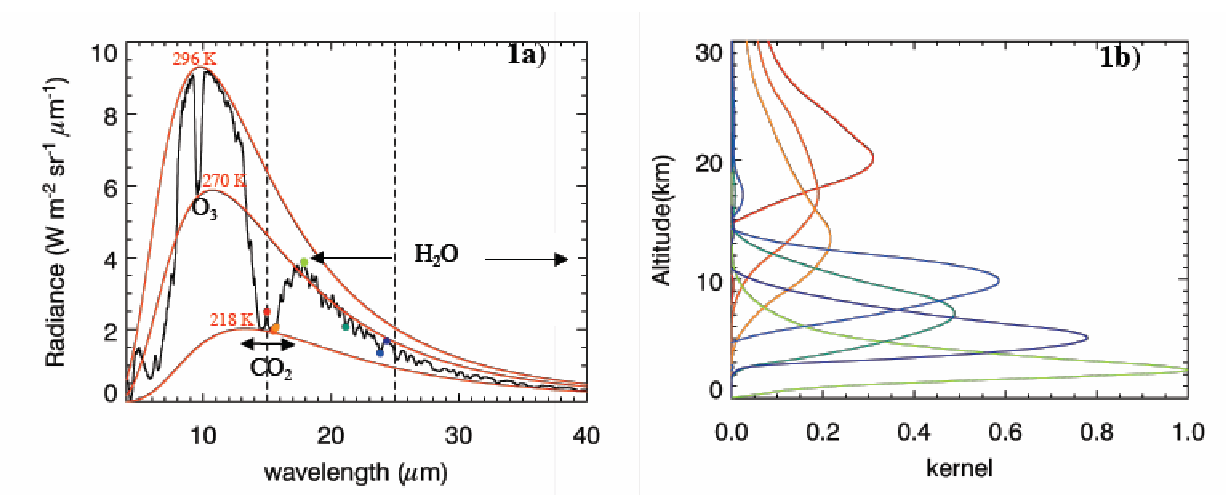
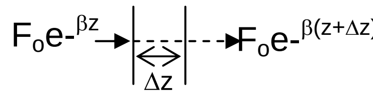


Figure 1a. A modeled top-of-atmosphere spectrum of nadir radiance at a spectral resolution similar to the proposed IMISR, ~75 nm. The CO_2 and O_3 absorptions bands are labeled, as is the extended region beyond 18 μm containing numerous pure-rotational water absorption bands. The red curves are Planck blackbody spectra at the indicated temperatures. **Figure 1b.** Normalized weighting functions at 7 discrete wavelengths in the 15-25 μm region. Top-of-atmosphere radiance in this spectral region contains information from the lower troposphere through the middle stratosphere. Colors refer to wavelength bands as indicated in **1a**).

Flux Divergence

- If more radiant energy enters a region than leaves it, radiant energy converted into other forms; usually temperature increase or other (e.g., photosynthesis).
- Consider amono-directional beam propagating along the *z*-axis in an optically homogeneous medium characterized by an absorption coefficient β:



- The difference between the radiant energy (per unit area) entering region and that leaving is: $F_o \exp(-\beta z) - F_o \exp(-\beta z + \Delta z)$
- Divide by Δz and take the limit as Δz → 0 to obtain the rate of energy conversion (transformation) per unit volume around a point:

$$-F_o \frac{d}{dz}\exp(-\beta z) = -\frac{dF}{dz} = F_o \kappa \exp(-\beta z)$$

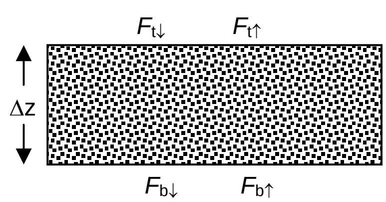
- Conservation of energy: net emitted (absorbed) energy equal to net loss (gain) of thermal energy (heat).
- The rate of energy transformation per unit volume is the negative of the spatial derivative of the irradiance, often called the flux divergence.
- The negative flux divergence is proportional to the local rate of temperature change under the assumption that radiant energy transformation results only in temperature increases:

$$-\frac{dF}{dz} = F_o \kappa \exp(-\kappa z) = \rho C_p \frac{\partial T}{\partial t}$$

- Note that the flux divergence is a product of two functions, one of which increases with increasing β, the other of which decreases; the maximum occurs for $\beta z = 1$ for fixed z > 0. (note similarity with weighting function.)
- In three dimensions: $-\nabla F = \rho C_p \frac{\partial T}{\partial t}$

1-D Flux divergence

- Downwelliing Irradiance: $F\downarrow$
- Upwelliing Irradiance: $F\uparrow$
- Net Flux: $F_{net} = F\uparrow - F\downarrow$



- Flux Divergence (emitted or absorbed): $\frac{F_{net,top} - F_{net,bottom}}{\Delta z} = \frac{(F_\uparrow - F_\downarrow)_{top} - (F_\uparrow - F_\downarrow)_{botton}}{\Delta z}$

Mie Lab

1. *Look at the Mie code. Write down the name of the subroutine that performs each of the four steps of "How a Mie code works" in the class notes. Find how many times the lines inside the inner loop are executed for steps 1, 2, and 3. How will these depend on size parameter?*

miecalc: Calculation of Mie coefficients a_n and b_n Nradii*Nterms
miecross: Calculation of the efficiencies (Q_{ext}, etc.) Nradii*Nterms
mieangle: Calculation of the phase function Nradii*Nterms*Nquad
miedist: Integration over the size distribution

Nradii is the number of radii in the size distribution integration, Nterms is the number of terms in the Mie series (number of a_n and b_n), and Nquad is the number of quadrature angles at which the phase function is computed.

The number of Mie series terms is Nterms ≈ x. The number of phase function angles is Nquad ≈ 2x. More angles are required to represent the phase function for larger size parameters because it has more structure due to higher order angular functions.
Thus the number of operations for the Mie cross sections are proportional to Nradii*x, while the number of operations for the Mie phase functions are proportional to Nradii*x^2.

2.

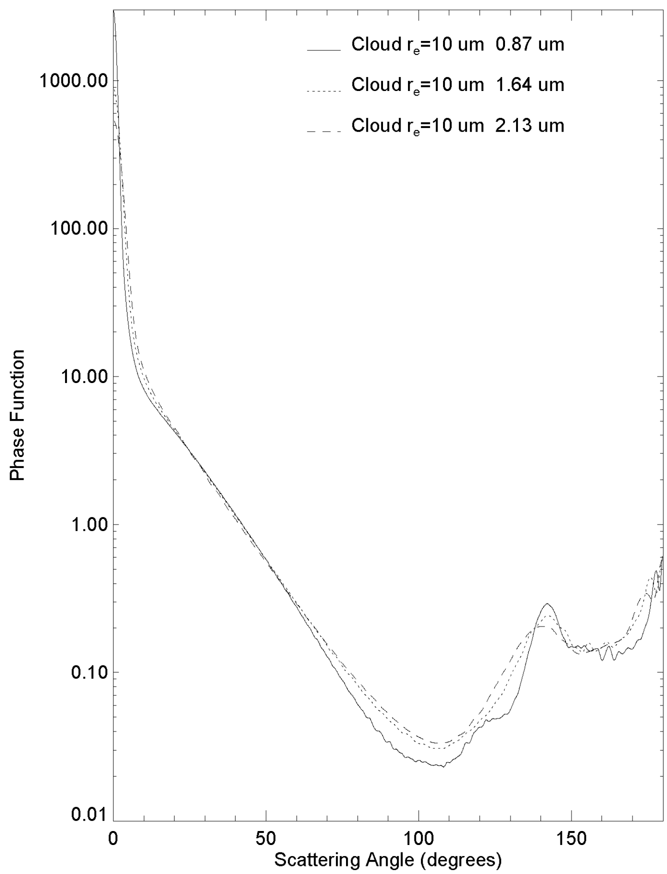
λ (μm)	m	r_{eff} (μm)	N (cm^{-3})	β (km^{-1})	ω	g	N_L
0.87	$1.330 - 3.25 \times 10^{-7}i$	10	100	48.00	0.99995	0.856	510
1.64	$1.317 - 7.91 \times 10^{-5}i$	10	100	49.58	0.99404	0.843	265
1.64	$1.317 - 7.91 \times 10^{-5}i$	5	100	13.07	0.99705	0.800	130
2.13	$1.296 - 3.96 \times 10^{-4}i$	10	100	50.51	0.9785	0.843	203
11.00	$1.155 - 9.82 \times 10^{-2}i$	10	100	38.87	0.4748	0.924	39
0.65	$1.56 - 0.01i$	0.7	200	0.321	0.8880	0.682	75

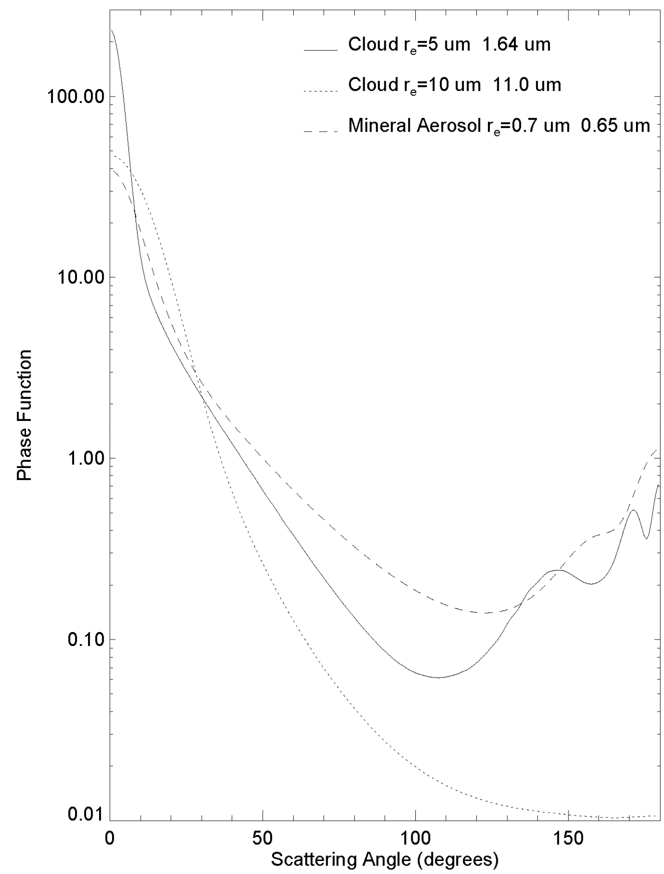
3.

λ (μm)	r_{eff} (μm)	β_{Mie} (km^{-1})	β_{GO} (km^{-1})	ω_{Mie}	ω_{GO}
0.87	10.0	48.00	45.24	0.99995	0.99996
1.64	10.0	49.58	45.24	0.99404	0.99468
1.64	5.0	13.07	11.31	0.99705	0.99734
2.13	10.0	50.51	45.24	0.97850	0.97981
11.00	10.0	38.87	45.24	0.47480	0.13619
0.65	0.7	0.32	0.23	0.88800	0.85926

Even though only the $\lambda = 0.87$ μm case is actually close to the geometric optics limit, the simple formulas for the extinction and single scattering albedo work quite well for the other solar wavelength cloud cases. The formulas do not work well for the aerosol case because the size parameter is much smaller than valid for geometric optics (the Mie regime resonant oscillation causes the actual extinction to be larger). The $\lambda = 11$ μm case is not in the geometric optics limit and not weakly absorbing, so it fails especially badly for the single scattering albedo.

4.





The three cloud phase functions on the first page are very similar, except the diffraction peak narrows and the rainbow and glory features become more distinct for the larger size parameters. (The small oscillations in the $\lambda = 0.87$ μm case are due to not having enough terms in the size distribution integration). The phase function for the $\lambda = 11$ μm cloud case shows very little scattering backwards due to absorption within the droplets. The mineral aerosol phase function has a lower, wider forward peak due to the small size parameter but a substantial backscattering peak from the high real part of the index of refraction.

The effective size parameter is $x_{eff} = 2\pi r_{eff}/\lambda$. The half power scattering angle is estimated from the phase function file.

λ (μm)	r_{eff} (μm)	x_{eff}	$P(0)$	$\Theta_{1/2}$ (degrees)	$\Theta_{1/2} x_{eff}$ (degrees)	$P(0)/x_{eff}^2$
0.87	10	72.2	2956	1.1	78	0.57
1.64	10	38.3	865	2.0	77	0.59
1.64	5	19.2	232	4.0	77	0.63
2.13	10	29.5	532	2.6	77	0.61
11.00	10	5.7	47.0	12.9	74	1.45
0.65	0.7	6.8	38.9	9.4	64	0.84

From diffraction theory, which applies to the geometric optics limit, we expect an inverse relation between the width of the forward scattering peak and the size parameter. The peak of the phase function should goes as x^2.The last two columns of the table show that these relationships hold for the solar wavelength cloud cases.

Explain why the asymmetry parameter is highest for the $\lambda = 11$ μm *cloud case even though the size parameter is lower than for the shorter wavelength cloud cases.*

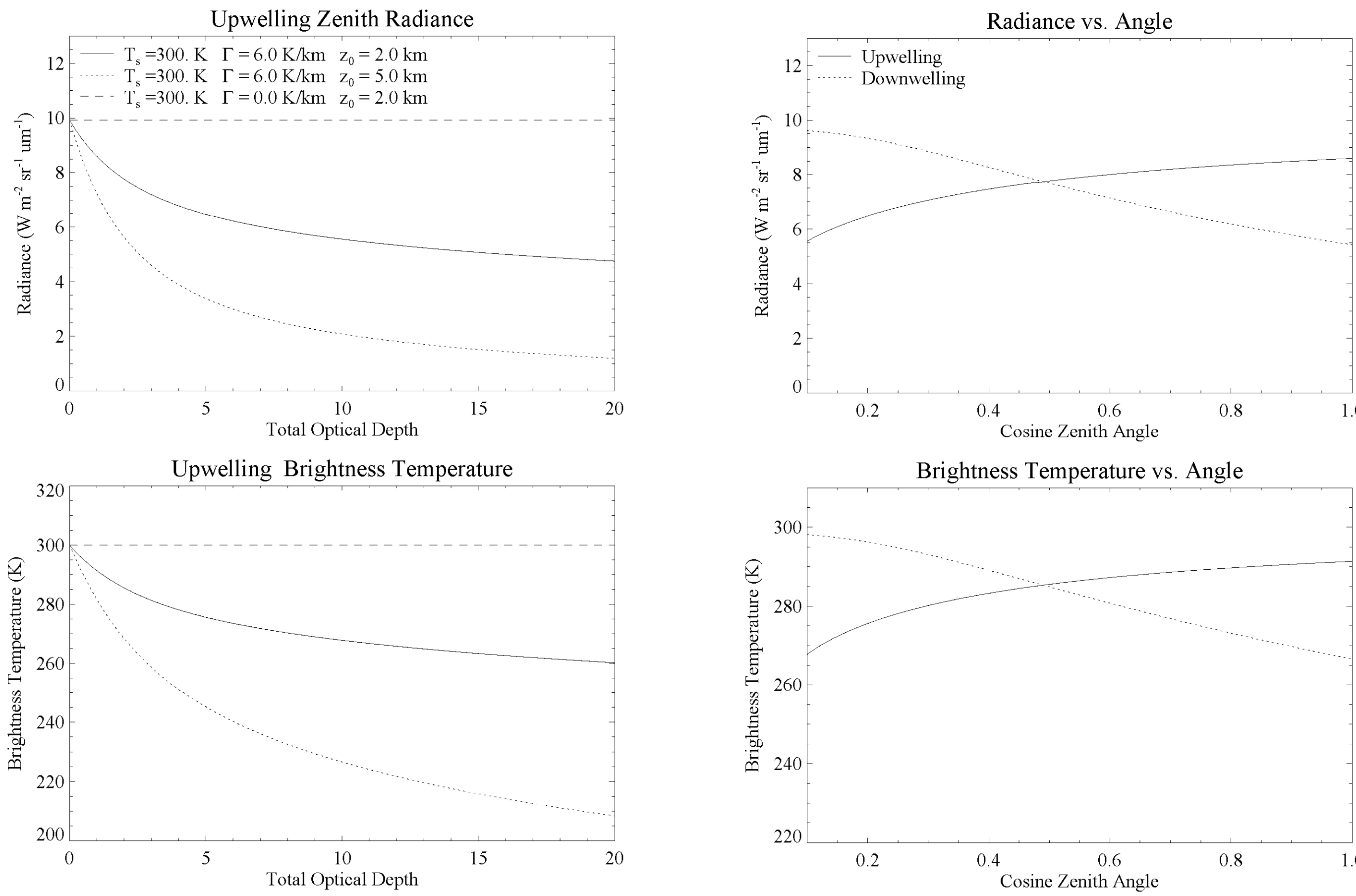
Light that penetrates significantly into the particle is absorbed, so most of the scattering is due to diffraction, which is around the forward direction.

5. The phase matrix at 125.2° is:

$$\begin{pmatrix} 0.0487 & -0.0244 & 0 & 0 \\ -0.0244 & 0.0487 & 0 & 0 \\ 0 & 0 & 0.0486 & 0.0239 \\ 0 & 0 & -0.0239 & 0.0486 \end{pmatrix}$$

Unpolarized light is of the form [*I, Q, U, V*] = [1, 0, 0, 0]. Light scattering is represented by multiplying the incident Stokes vector by the phase matrix. The resulting normalized scattered Stokes vector is [1,-0.46, 0, 0]. The degree of linear polarization is $\sqrt{Q^2 + U^2}/I = 0.46$.

Thermal RT lab



3b) *Explain the dependence of radiance on total optical depth and the differences between the three cases.*

The upwelling radiance decreases with increasing optical depth for the atmospheric profiles having decreasing temperature. As the optical depth increases, the emission from the surface and lower atmosphere layers is obscured (little transmission), and the upwelling radiation comes from emission higher in the atmosphere. Since higher layers are colder in this atmosphere, the Planck function is less. Thus the high radiance from the warmer layer emission is replaced by lower radiance from the colder layer emission. Why don't the cold highest layers of the atmosphere cause the upwelling radiance to be very low? They normally have very little optical depth, so their emissivity is small and they transmit most of the upwelling radiation from below. So the upwelling radiance is emitted from a broad range where the transmission is less than one but above zero. The isothermal case has no dependence on optical depth because the Planck function is constant, regardless of the level in the atmosphere from which the emission originates. The larger scale height case gives lower upwelling radiance because the emission comes from higher (colder) in the atmosphere. The larger extinction scale height means that the moderate transmission region is a greater altitudes.

3c) *For the first two cases calculate the heights and optical depths where the atmospheric temperature matches the brightness temperature for a total optical depth of 10.*

i) z = 5.37 km and τ = 0.68 ii) z = 12.24 km and τ = 0.86.

Even though the equivalent heights are quite different, the corresponding optical depths are similar, around τ = 1. (consider weighting functions)

4b) *List the upwelling and downwelling brightness temperatures for μ = 1 and μ= 0.3. Briefly explain the radiance behavior with μ.. What is this effect called for the upwelling and downwelling cases?*

Up μ = 1.0 T_b = 291.4 K; μ = 0.3 T_b = 280.2 K
Down μ = 1.0 T_b = 266.6 K; μ = 0.3 T_b = 293.1 K

Limb darkening for upwelling. Limb brightening for downwelling.

The slant paths have longer optical paths, which means that the emission comes from height levels closer to the observer, as compared to straight (μ = 1) viewing. For upwelling radiance the emission for slant paths originates from colder layers, hence limb darkening. For downwelling radiance the emission for slant paths originates from warmer layers, hence limb brightening.

5a) The angular integration to determine irradiance is performed with quadrature:

$$F = 2\pi \sum_{j=1}^{n} w_j \mu_j L(\mu_j)$$

τ_{tot} = 1.0: F = 25.10 W m^{-2} μm^{-1}.
τ_{tot} = 5.0: F = 18.31 W m^{-2} μm^{-1}

5b) *For the two cases in a) find the approximate $\bar{\mu}$ that gives the same irradiance.*
The single angle or diffusivity approximation is $F = \pi L(\bar{\mu})$. $D = 1/\bar{\mu}$ is called the diffusivity factor. For upwelling irradiance for the two cases the best fitting angle is τ_{tot} = 1.0 $\bar{\mu}$ = 0.59 and τ_{tot} = 5.0 $\bar{\mu}$ = 0.62. Both of these are close to $\bar{\mu}$ = 0.6, and so a diffusivity factor of D = 5/3 is often assumed.