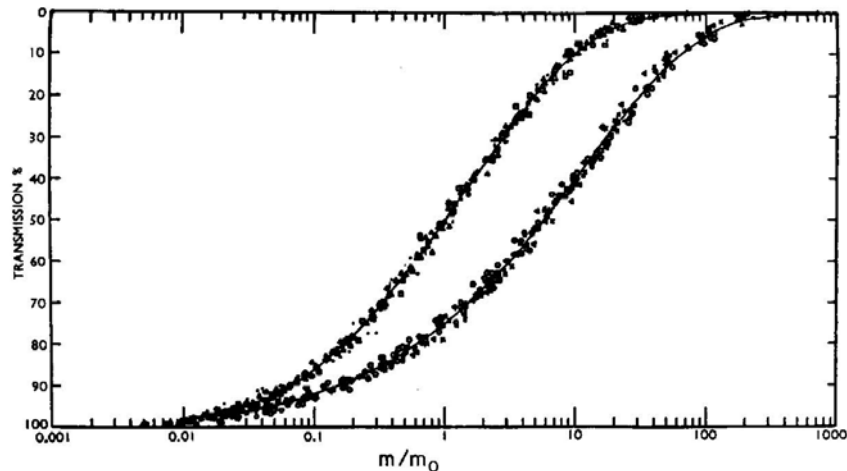


Band Models

Band models are simple expressions for the mean transmission over a spectral band with many lines. Some generally available radiative transfer codes, such as MODTRAN and MDTERP, use band models.

- Regular (for example, the *Elsasser* model) band model assume evenly spaced, identical strength lines.
- Random band models (for example, Goody or Malkmus) assume n randomly spaced lines in $\Delta\nu$ band (n lines of mean distance δ so $\Delta\nu = n\delta$). The lines are independent and have identical shapes. Probability density of strength of i 'th line is $p(S_i)$. Different $p(S)$ are assumed in different models.
- Approach: Derive mean transmission by multiplying transmissions of each line at a particular ν , and also integrating over probability distributions of line positions ν_i and line strengths S_i for each line.
- Goody model has exponential line strength distribution.; Malkmus model has a higher probability of weak lines.

How well does the Goody random band model work?



Comparison of the random band model with laboratory measurements in water vapor bands. The left curve is for a pressure of 740 Torr and the right curve is for a pressure of 125 Torr. m is the water vapor amount, and m_0 is the amount of water vapor that gives a transmission of 0.5 at 740 Torr. The symbols indicate the particular water vapor vibrational bands from 6.3 μm to 1.1 μm . [Goody & Yung, Fig. 4.18]

Problems with band models:

- Curtis-Godson approximation for inhomogeneous paths is not accurate when absorber and pressure are not correlated (e.g. O_3).
- Band models can't handle scattering -they assume source function is constant, but scattering source function depends on radiance.
- A heating rate calculation with band models goes as N^2 where N is the number of layers.

***k*-Distribution Method**

In general, the mean band transmission is written: $T_{\Delta\nu} = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-k_\nu u) d\nu$

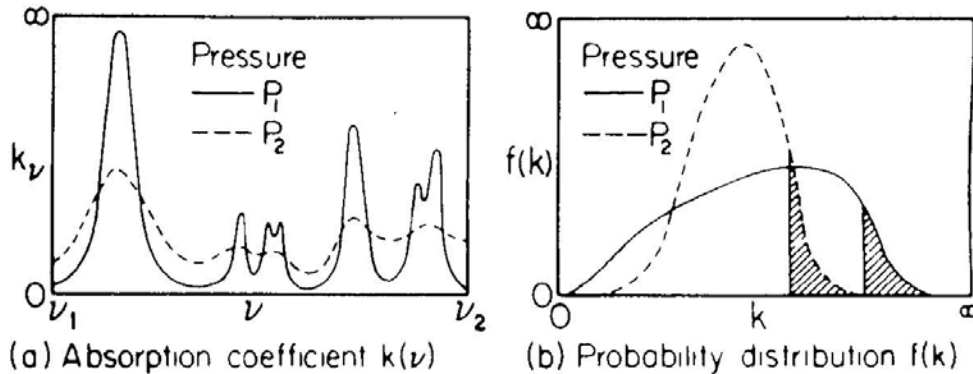
k is wavenumber, u is absorber amount.

To perform the integration, the order (in wavenumber) in which the exponentials are summed doesn't matter. Instead of summing from low to high wavenumber, we can sum from low k to high k .

Rearrange the integral to be over the absorption coefficient k :

$$T_{\Delta\nu} = \frac{1}{\Delta\nu} \int_{\Delta\nu} \exp(-k_\nu u) d\nu = \int_0^\infty \exp(-ku) f(k) dk$$

$f(k)$ is the fraction of the spectral band with absorption between $k \rightarrow k+dk$, or the probability density function of absorption ($\int f(k) dk = 1$); $f(k)$ is smooth – only few terms in needed in integration sum.

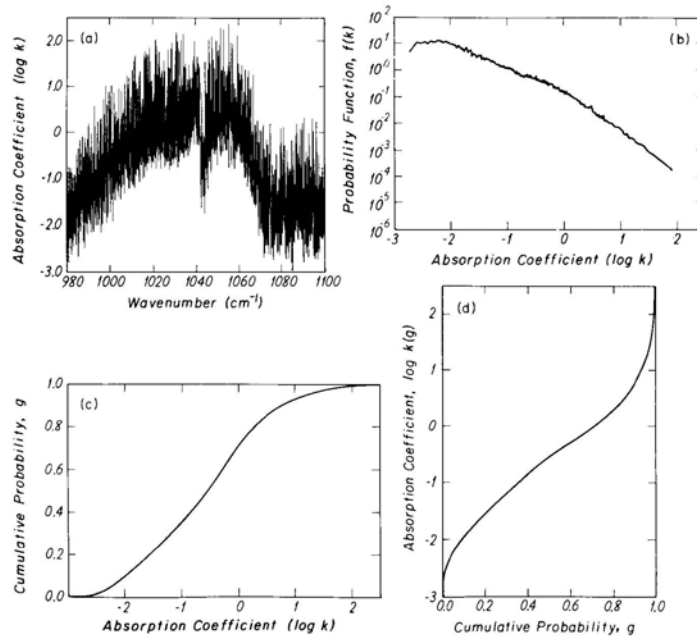


A schematic illustration portraying the essence of the k -distribution method. (a) A schematic of absorption line spectra at two different pressures. (b) The two probability density functions $f(k)$ associated with (a). The shaded area depicts the strongest absorption (i.e. largest k 's) which come from the same spectra regions. [from G. L. Stephens]

It is simpler to integrate over the cumulative probability function: $g(k) = \int_0^k f(k) dk$; $g(k)$

is the fraction of the spectrum with absorption coefficient below k .

Now the mean band transmission can be written: $T_{\Delta\nu} = \int_0^1 \exp\{-k(g)u\} dg$



(a) Absorption coefficient k in units of $\text{cm}^{-1} \text{atm}^{-1}$ as a function of wavenumber with a resolution of 0.05 cm^{-1} in the $9.6 \mu\text{m}$ ozone band ($p = 30 \text{ mb}$; $T = 200 \text{ K}$). (b) The probability function $f(k)$ of the absorption coefficient. (c) The cumulative probability distribution function of $f(k)$ plotted as a function of k . (d) Same as (c), except that values of the absorption coefficient are expressed as a function of g . [Liou (1992), Fig. 2.15]

Discrete k -distribution Sums

- Discrete sum is accurate because $k(g)$ is smooth.
- Divide g range from 0 to 1 up into intervals Δg_j over which $k(g)$ can be approximated to be constant:

$$T_{\Delta v} = \sum \exp\{-k(g_j)u\} \Delta g_j$$

- g_j is the probability interval: fraction of spectrum represented by the mean absorption coefficient $k(g_j)$.
- One “ k ” in sum corresponds to Beer’s law: monochromatic, gray (no spectral variation), or low optical depth.
- Compute $g(k)$ directly from line-by-line k_v spectrum. Sort k_v , start of array is $g = 0$, end (highest k) is $g = 1$. Divide sorted array into Δg_j intervals, average each to get k_j .

Inhomogeneous Paths -Correlated k-distribution

Each pressure and temperature along a path has a unique k_v spectrum. A correlated k -distribution sorts each $k_v(p, T)$ spectrum independently to make k -distributions $k(g; p, T)$ for each p and T . (Note: this T is temperature, not transmission!) In practice, discrete k -distributions $k_j(p_l, T_m)$ are made for a set of pressures p_l and temperatures T_m and interpolated in between.

Use the k_j 's for all L layers as in monochromatic calculations:

$$T_{\Delta\nu}(u) = \sum g_j \exp(-\sum k_j \Delta u_l) \quad T_{\Delta\nu}(u) = \sum \Delta g_j \exp\left\{-\sum k(g_j) u_l\right\}$$

Justification:

- Absorption spectra for a gas at different pressures are correlated.
- Correlated k assumption is exact when each k -distribution $k(g; p, T)$ sorts the wavenumbers in exactly the same order, e.g., a single line or evenly spaced lines.
- In reality, sorting all layer spectra on total optical depth does not give monotonic absorption coefficient vs. cumulative probability.
- The real justification is that correlated k errors are $< 1\%$ in typical atmospheric radiative transfer computations.

Using a k -distribution

Do a weighted sum of monochromatic calculations (one for each “ k ”):

- For each layer l interpolate k -distribution absorption coefficient to layer pressure and temperature to get $k_j(z_l)$.
- Multiply $k_j(z_l)$ by absorber amount in layer and add to particle scattering optical depth in layer: $\Delta\tau_j(z_l) = k_j(z_l)\Delta u(z_l) + \tau_{\text{sca}}(z_l)$
- Solve monochromatic radiative transfer equation using the layer optical depths to get radiance or irradiance (e.g. $F_j(z_l)$). Use the band integrated Planck function or solar irradiance for the source function.
- Weight result for each k in sum by Δg_j : $F_{\Delta\nu}(z_l) = \sum_j \Delta g_j F_j(z_l)$ to get the spectral band integrated irradiance profile.