Reading: Bohren and Clothiaux 5.4 and Ch. 2 (next week); Thomas & Stamnes 2.7-2.8 and 5.4

Supplemental Reading: Gaussian Quadrature and Its Application to Infrared Radiation (http://atoc.colorado.edu/~pilewskp/IR_rad_quad.pdf)

Discrete Multilayer Thermal Radiative Transfer Solution

Computer algorithms can integrate the thermal Radiative Transfer Equation by iterating the single layer solution. This assumes the layers are thin enough so temperatures are close to the mean layer temperature.

- Divide atmosphere into N discrete layers
- For upwelling radiance:
  - Start at blackbody surface $L_{N+1}(\mu) = B(T_s)$
  - Iterate going upward (i decreasing):
    $L_i(\mu) = e^{-\Delta\tau/\mu} L_{i+1}(\mu) + B(T_i)\left[1 - e^{-\Delta\tau/\mu}\right]$
- For downwelling radiance:
  - Start at Top-of-Atmosphere $L_1(\mu) = 0$
  - Iterate going downward (i increasing):
    $L_{i+1}(\mu) = e^{-\Delta\tau/\mu} L_i(\mu) + B(T_i)\left[1 - e^{-\Delta\tau/\mu}\right]$

The radiance exiting one computational layer is the transmitted radiance plus the emitted radiance. This equation is used to propagate a ray at angle μ through many layers.

Hemispheric irradiance is obtained from an integral over solid angle:

$$F^{\uparrow} = 2\pi \int_0^1 \mu \left\{ e^{-\tau_0/\mu} B(T_0) + \int_0^{\tau_O} B(T) e^{-\tau'/\mu} \frac{d\tau'}{\mu} \right\} d\mu$$

The integration may be expressed with exponential integral functions.
In practice, the angular integration is performed with quadrature:

$$F^{\uparrow} = 2\pi \sum_{j=1}^{n} w_j \mu_j L(\mu_j)$$

Four-point Gauss quadrature gives accurate results for integration over μ:

n = 4: μ_1 = 0.069432 w_1 = 0.173927
μ_2 = 0.330009 w_2 = 0.326073
μ_3 = 0.669991 w_3 = 0.326073
μ_4 = 0.930568 w_4 = 0:173927

The single angle or diffusivity approximation is $F = \pi L(\bar{\mu})$, and $D = 1/\bar{\mu}$ is called the diffusivity factor. A diffusivity factor of $D = 5/3$ is often assumed.