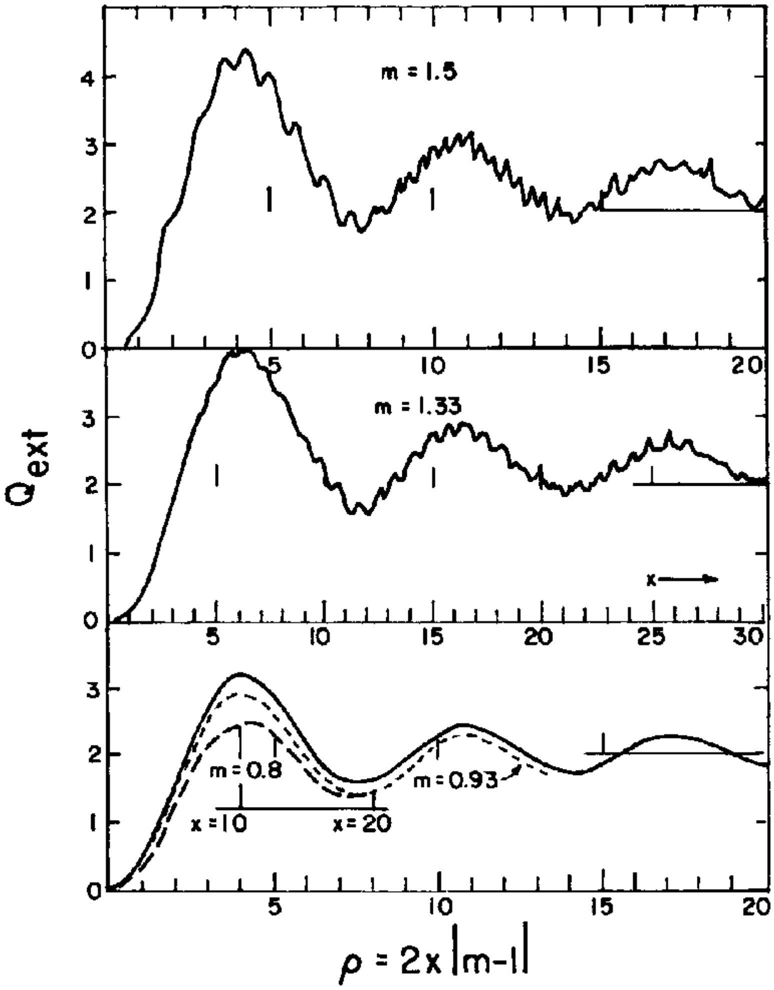
Anomalous Diffraction Theory (ADT)

Simple scattering theory - explains main Q_{ext} oscillations.
ADT applies to limits: $x >> 1$ and $m - 1 << 1$ so no little refraction or reflection.
phase difference: $\rho = 2x(m\text{-}1)$.
ADT integrates sum of incident and transmitted E field in forward direction
Uses optical theorem $C_{ext} = \frac{4\pi}{k^2}\Re\{S(0°)\}$ to obtain extinction cross section, and bulk absorption coefficient to obtain absorption cross section.

Oscillations in Q_{ext} due to constructive and destructive interference of diffracted and transmitted waves.

For non-absorbing spheres ADT gives: $Q_{ext} = 2 - \frac{4}{\rho}\sin\rho + \frac{4}{\rho^2}(1 - \cos\rho)$



Extinction curves computed from Lorenz-Mie theory for m=1.5, 1.33, 0.93, 0.8. The abscissa is $\rho = 2x(m\text{-}1)$ and is common to the upper two Mie curves as well as to the bottom anomalous diffraction theory (ADT) curves [van del Hulst, 1957; Stephens, 1994]

Angular dependence form diffraction

ADT provides extinction and absorption, but not phase function.

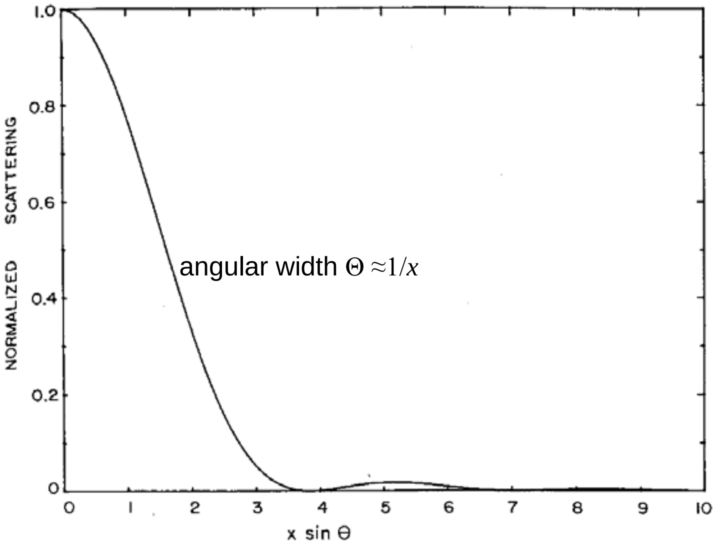
In geometric optics limit (x >> 1) light may be treated as rays, except for Fraunhofer diffraction around a particle.

Babinet's principle - diffraction pattern is the same from an aperture as for opaque particle of same size.

For sphere (circular aperture) the diffraction pattern is:

$$I(\Theta) = \frac{I_0}{k^2 R^2} \frac{x^4}{4} \left[\frac{2 J_1(x \sin\Theta)}{x \sin\Theta} \right]^2$$

Diffraction peak in phase function (for geometric optics limit): the angular width of the forward scattering peak (defined at half the maximum, at P(0°)) occurs at $\Theta \approx 1/x$. The height at $P(0°) \propto x^2$.



Some non-spherical computational methods

- Ray Tracing
- Discrete Dipole Approximation
- T–Matrix Method

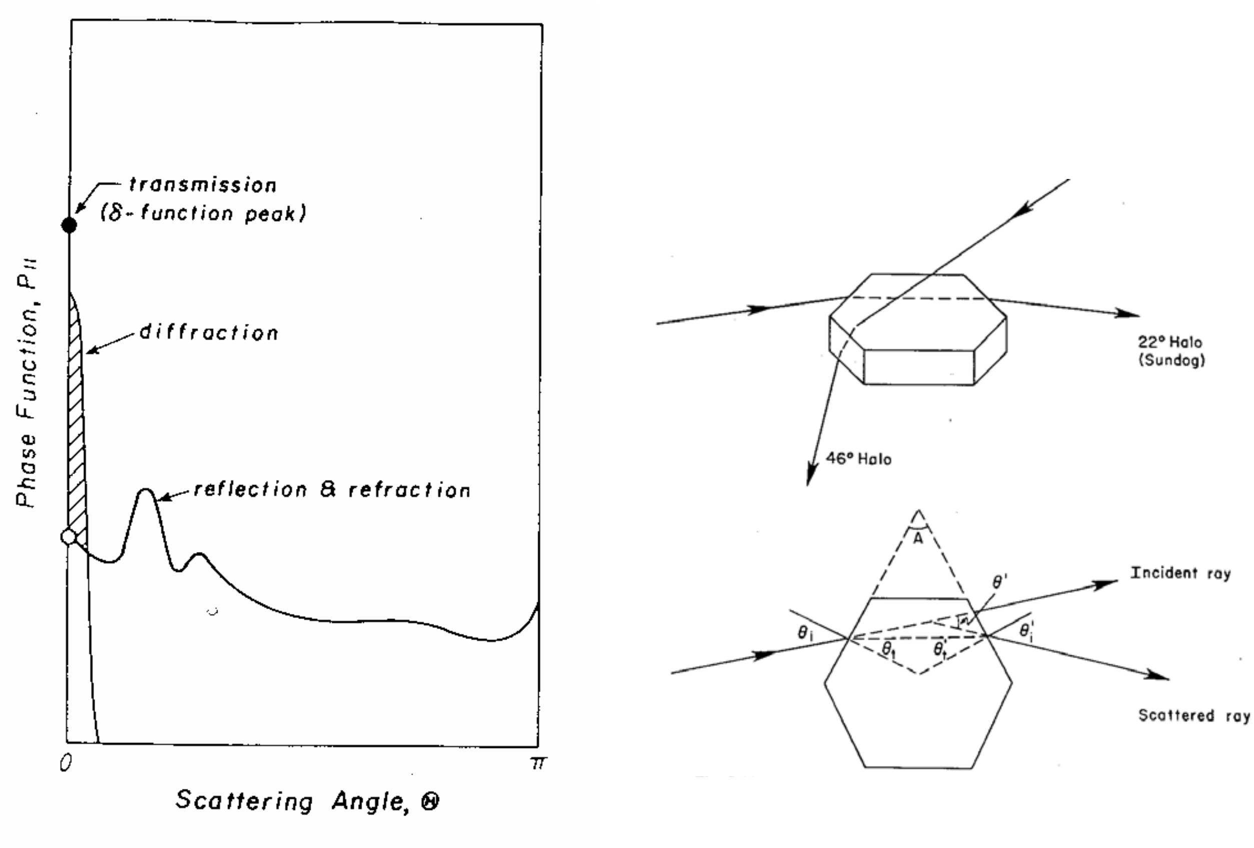
Ray Tracing

In the geometric optics limit ray optics provide an accurate scattering method for non-spherical particles.

Ray optics consists of two parts:

1) Diffraction theory for the forward scattering peak,
2) Ray tracing using Fresnel reflection and transmission formulas.

Many rays incident on particle are followed to determine what fraction scatter in each direction or are absorbed.



Left: schematic representation of the components of the phase function for randomly oriented hexagonal ice crystals. Right: geometrical reflection and refraction by hexagonal crystals.

Fresnel Reflection and Transmission

Plane wave incident on planar dielectric surface. Solve for reflection and transmission using boundary conditions from Maxwell's equations (tangential components of fields are continuous).

Snel's Law: $\sin\theta_i = m\sin\theta_t$

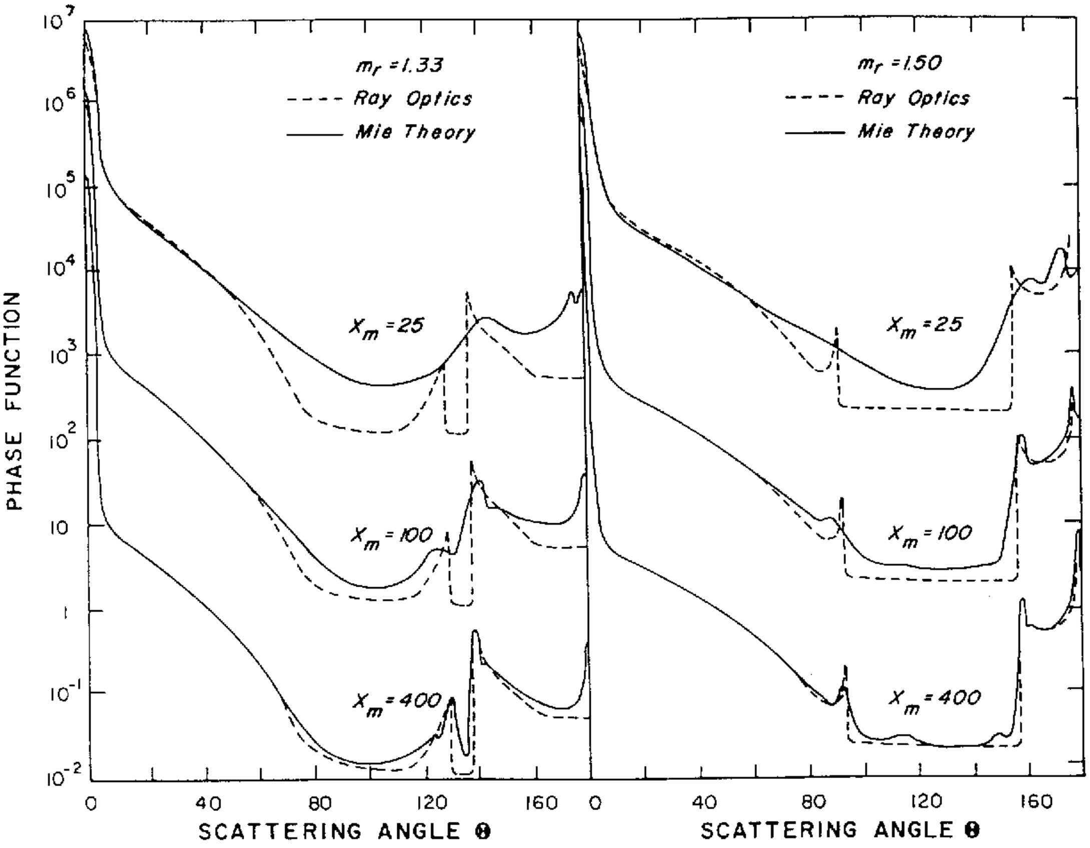
Fresnel formula for polarized reflection *amplitude* coefficients:

$$r_\perp = \frac{\cos\theta_i - \sqrt{m^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{m^2 - \sin^2\theta_i}} \qquad r_\parallel = \frac{\sqrt{m^2 - \sin^2\theta_i} - m^2\cos\theta_i}{\sqrt{m^2 - \sin^2\theta_i} + m^2\cos\theta_i}$$

Reflection and transmission coefficients for irradiance:

$$R_\parallel = \left|r_\parallel\right|^2 \text{ and } R_\perp = \left|r_\perp\right|^2 ;\, T_\parallel = 1 - R_\parallel \text{ and } T_\perp = 1 - R_\perp$$

Ray tracing compares well with Mie theory for large size parameters:



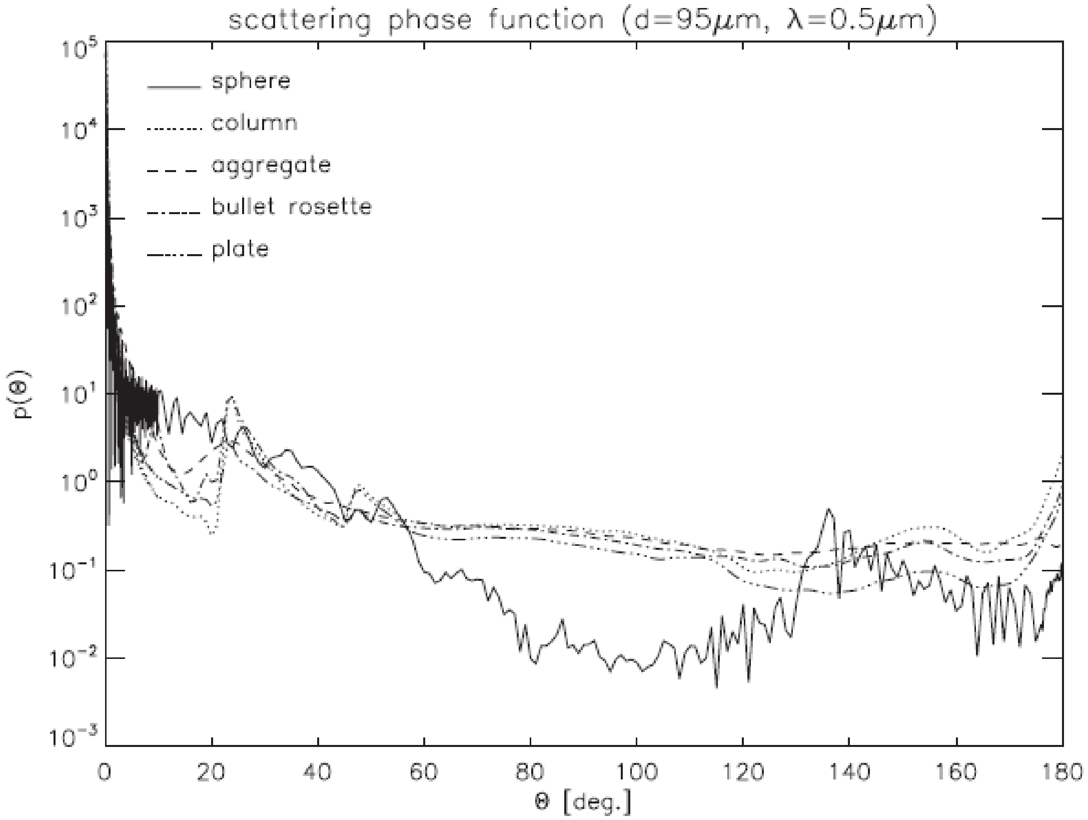


Figure 3.15: Comparison of spherical and nonspherical phase functions. This figure was provided by André Ehrlich (University of Mainz) on the basis of computations by Yang et al. (2000).

Yang, P., Liou, K., Wyser, K., and Mitchell, D.: Parameterization of the scattering and absorption properties of individual ice crystals, J. Geophys. Res., 105, 4699–4718, 2000.

Discrete Dipole Approximation

- A numerical method for scattering from any shape particle that is not too large ($x < 5$). See review paper by Draine and Flatau, J. Opt. Soc. Am. A, **11,** 1491.
  - Particle is divided into dipoles of size *d* small compared to wavelength.
  - The field at one dipole is determined by all other dipoles.
  - For a particular incident *E* field, a linear system of equations may be solved for the dipole moments.
  - The far field scattering properties are calculated from the dipole moments.
- Finite-Difference Time Domain (FDTD) is a competing method that is also used for small non-spherical particles.
- In both DDA and FDTD the number of dipoles or elements is proportional to the particle volume → quickly becomes computationally prohibitive.

T–Matrix Method

- Expand incident field into vector spherical harmonics (similar to Mie theory).
- Resulting equations of the expansion coefficients of the incident and the scattered electric fields are linear.
- The matrix realizing the transformation of the expansion coefficients from the incident to the scattered fields is called the *T*- matrix. (A kind of a Müller matrix).
- Can be used for any scattering property of nonspherical particles
- Highly accurate and fast, public codes available, size parameter may exceed 100
- The *T*-matrix is independent of the incident and scattered fields. It only depends on the shape of the particle, the size parameter, the refractive index, and the orientation of the particle.
- It needs to be simulated only once and is then applicable for any direction of incidence and scattering angle.
- Large computation time