Coherent Vortices and Tracer Transport

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Summary

Geophysical flows are characterized by the presence of coherent vortices, localized concentrations of energy and vorticity that have a lifetime much longer than the local turbulence time (sometimes called the eddy turnover time).

In the ocean, coherent vortices, or eddies, are ubiquitous features whose size varies between several to a few hundred kilometers, and that account for a large portion of the ocean turbulent kinetic energy [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. The presence of vortices can be revealed in various ways. Vortices at the ocean surface imprint their signature on the sea surface height and can be tracked by satellite, while floats with looping trajectories can help revealing the presence of vortices at depth.

Coherent vortices significantly affect the dynamics and the statistical properties of ocean flows, with important consequences on transport processes. In this contribution, we shall briefly review some of these issues, focusing on the simplified conceptual model provided by two-dimensional turbulence.

1 Coherent Vortices and Background Turbulence

The dynamics of vortex-dominated geophysical flows can be simulated by adopting the overly simplified configuration of two-dimensional, barotropic turbulence, where the motion is purely horizontal and vertical derivatives vanish. The dynamics of two-dimensional turbulence is described by the vorticity equation

$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega = F + D$$

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where $\omega(\mathbf{x}, t) = \partial v/\partial x - \partial u/\partial y$ is vorticity, $\mathbf{u} = (u, v)$ is the fluid velocity, $\mathbf{x} = (x, y)$ is space and t is time. The terms F and D represent forcing and dissipation respectively.

The dynamics of two-dimensional turbulence is characterized by the spontaneous emergence, and subsequent dominance, of a population of strong coherent vortices that concentrate most of the energy and vorticity of the flow [6, 19, 18]. In past years we have advocated the view that two-dimensional turbulence can be pictured as a two-component fluid: a sea of coherent vortices immersed into a background turbulence that is quite Kolmogorovian. This two-component view forms the basis of how we interpret Lagrangian (and Eulerian) measurements and how we infer flow properties from them [20, 21, 22].

An important issue is how we identify the two components. Until now, the best way to identify vortices is found to be the direct identification by some vortex census algorithm based on the analysis of local vorticity patches in physical space. A variety of such methods exists [23, 24, 25, 26, 27, 28]; all of them require the knowledge of the full vorticity field. A simplified version of a vortex census, which requires the knowledge of just a few Eulerian time series and provides the gross features of the vortex statistics such as the vortex density and the average vortex size, has also been proposed [22].

Although coherent vortices are local vorticity concentrations, their effects are non-local: The velocity field generated by a coherent vortex is non-local as it extends to large distances from the vortex center, well beyond the region where vorticity is significant. The range where the effect of the vortex on the velocity field is significant depends on the vortex shape and on the degree of baroclinicity: Barotropic vortices extend their influence to far distances, while baroclinic lenses (such as Meddies) have a shorter range of influence. Indeed, the Green's function associated with a barotropic (point) vortex is proportional to $\log(r)$, where r is the distance from the core of the vortex. For a baroclinic (point) vortex, the Green's function goes as $1/r$. Therefore baroclinic vortices have a shorter range of influence than barotropic ones [29]. In terms of the velocity field (and particle dispersion), the two-component view of mesoscale turbulence should not be seen as a purely spatial decomposition of space into separate vortex and non-vortex areas, but rather as the superposition of two dynamical components which can simultaneously act at the same spatial position.

The far-field influence of coherent vortices can be seen in the probability distribution function (PDF) of the velocity. At high Reynolds numbers, when vortices are intense and have sharp profiles, velocity PDFs in barotropic turbulence have non-Gaussian tails indicating that high velocities are more probable than would be the case for a Gaussian field [21, 30]. This non-Gaussianity has been previously discussed in the context of point vortices, which can be thought of as a simplified model of vortex dominated flows at very large Reynolds number [31, 32, 33]. In this context, it has been shown that small velocities have a Gaussian distribution but the PDF has a non-Gaussian tail related to the slow decay with distance of the velocity induced by a single vortex. Convergence to a Gaussian PDF is obtained only in systems with an extremely large number of vortices, orders of magnitude more than exist in the ocean [33].

Float trajectories in the North Atlantic [30] and in the Adriatic Sea [34, 35], indicate that velocity PDFs are non-Gaussian. Typically, they have larger kurtosis than a normal distribution: they have a Gaussian-like core and non-Gaussian tails for high velocities. Similar results have been found from mid-latitude fluid particle trajectories along isobaric surfaces in a simulation of the Atlantic Ocean dynamics at high resolution [36]. Note that we are here referring to either Eulerian or Lagrangian velocity PDFs under the assumption that Lagrangian particles sample the whole domain. In this case, in fact, Lagrangian velocity PDFs in the ocean must converge to the Eulerian ones. This similarity in velocity PDFs between float data, ocean GCMs, simplified turbulence models, and point vortex systems suggests that the non-Gaussian nature of the velocity PDFs is due to the vortex component of ocean mesoscale turbulence.

2 Dynamics of Lagrangian Tracers

The Lagrangian equation of motion for an individual fluid particle moving in a two-dimensional flow is

$$\frac{\mathrm{d}\mathbf{X}_i}{\mathrm{d}t} = \mathbf{U}_i(t) = \mathbf{u}(\mathbf{X}_i(t), t) \tag{1}$$

where $\mathbf{X}_i(t)$ and $\mathbf{U}_i(t)$ are the Lagrangian position and velocity of the ith particle, and $\mathbf{u}(\mathbf{X}_i, t)$ is the Eulerian velocity at the particle position. In this equation, we do not equate force to mass times particle acceleration, but rather particle velocity to the push of the flow. This happens because the particle is assumed to have negligible size and vanishing inertia with respect to the advecting fluid, i.e., to be a fluid element. When particles have finite size and/or non-vanishing inertia, the equations of motion become more complicated, see e.g. [37, 38] for a discussion of the dynamics of inertial and finite-size particles in vortex-dominated flows.

Numerical simulation of barotropic and of baroclinic (stratified) quasi-geostrophic turbulence and of point-vortex systems indicate that the cores of coherent vortices are associated with islands of regular (non-chaotic) Lagrangian motion that trap particles for times comparable with the vortex lifetime [39], and that vortices are characterized by a strong impermeability to inward and outward particle fluxes, see e.g. Elhmaidi et al. [40] or Provenzale [38] for a review. Particles can have more complex behavior and can eventually migrate from inside to outside of a vortex or vice versa only when highly (and relatively rare) dissipative events take place, as the deformation of a vortex due to the interaction with a nearby vortex, or the formation of a filament.

For this reason, an initially inhomogeneous particle distribution becomes homogeneous only on a very long time scale, which is determined by the typical lifetime of the vortices rather than by their typical eddy turnover time.

The trapping behavior of coherent vortices can be rationalized in terms of potential vorticity (PV) conservation [41]. For an ideal fluid with irrotational external forcing PV is conserved. When some little dissipation and/or rotational forcing is acting on the fluid, as it usually happens, PV is not conserved. If the PV-changing effects are small, PV is quasi-conserved. This means that in regions where PV changes slightly, the particles will be able to shift from one PV surface to another. However, strong PV gradients are much more difficult to overcome, as the change in PV that the particle should achieve to climb (or descend) the gradient may be too large compared to the effect of the forcing and dissipation present in the system. As a result, strong PV gradients can act as transport barriers. This is the main physical reason why intense jets, associated with strong PV gradients, can act as efficient barriers to transport. The same happens for isolated vortices: Vortex edges act as barriers to transport because vortices are regions of anomalous potential vorticity, usually embedded in a background where PV oscillates around a reference value with low variance. The vortex edges are therefore characterized by a large potential vorticity gradient, which fluid particles can rarely cross. This behavior is clearly illustrated by the dynamics of the stratospheric polar vortex over Antarctica [42].

Another important effect of coherent vortices concerns the convergence of Lagrangian time-averages. Lagrangian particles can have a very long memory when coherent structures, whose lifetime is long compared to other time scales in the problem, are present. For instance, if a Lagrangian particle is initially released in the background turbulence outside vortex cores, it will move around without entering any of the vortex cores present in the turbulent flow, until, in a quite rare event such as the formation of a new vortex, the particle will get trapped inside a newly forming vortical structure. From that moment on, the particle will stay inside the vortex for times comparable with the vortex lifetime.

The above example indicates that the temporal convergence of the statistical properties of a set of Lagrangian trajectories can take place on rather long timescales, related to the lifetime of the coherent structures. Of course, ensemble averages over a large number of homogeneously distributed Lagrangian particles do not suffer from this problem and they usually give a more complete picture of the flow. This illustrates the fact that ergodicity (i.e., equivalence of time and ensemble averages) is reached only on very long times, if ever, for Lagrangian statistics of particles moving in vortex-dominated flows, as discussed by Weiss et al. [33] for point vortices and by Pasquero et al. [22] for the vortices of two-dimensional turbulence. An interesting question, then, concerns the trade-off between the number of particles required to provide a meaningful picture of the flow (i.e., a correct estimate of the statistical properties of the flow) and the length of the trajectories. This issue has been discussed in some detail in [22], together with the comparison between Lagrangian and Eulerian second-order statistics (i.e., spectra and decorrelation times).

3 Lagrangian Dispersion in Vortex-Dominated Flows

Lagrangian particles in a Gaussian, homogeneous, stationary and uncorrelated velocity field undergo a Brownian random walk. Under such conditions, the second-order moment of the distribution of particle displacements grows linearly with time:

$$A^2(\tau; t_0) \equiv < (\mathbf{X}_i(t_0 + \tau) - \mathbf{X}_i(t_0))^2 >= 2K\tau \tag{5}$$

where K is the dispersion (or diffusion) coefficient. Here, $\mathbf{X}_i(t)$ is the position of the ith particle at time t, and the angular brackets denote an ensemble average over all particles. The function $A^2(\tau, t_0)$ measures the absolute (or single-particle) dispersion. For a statistically stationary flow, the absolute dispersion A^2 does not depend on the starting time t_0. Relaxing any of the above assumptions (Gaussianity, homogeneity, stationarity, lack of temporal and spatial correlations) can significantly alter the dispersion law described above.

On short timescales, in particular, the Brownian dispersion law is modified by spatial and temporal correlations in the advecting flow, which induce Lagrangian velocity correlations over a substantial time range. The velocity autocorrelation function for an individual particle (labeled by the index i) is defined as

$$R_i(\tau) = \frac{\overline{(\mathbf{U}_i(t) - \overline{\mathbf{U}}_i) \cdot (\mathbf{U}_i(t+\tau) - \overline{\mathbf{U}}_i)}}{\sigma_i^2} \ , \tag{3}$$

where $\mathbf{U}_i(t)$ is the velocity of the ith particle at time t, $\overline{\mathbf{U}}_i$ and σ_i^2 are the mean and variance of the velocity of the ith trajectory, and the overbar indicates an average over time t. Hence, $R_i(0) = 1$ and $R_i(\tau)$ goes to zero for large τ, when the particle velocity loses memory of its initial value. The flow field as a whole is characterized by the ensemble-averaged velocity autocorrelation function, $R(\tau)$, defined by averaging over all trajectories. One simple measure of the memory of Lagrangian particles is the Lagrangian integral time, defined as

$$T = \int_0^\infty R(\tau) \mathrm{d}\tau. \tag{4}$$

Over times much shorter than the Lagrangian integral time, the velocity is almost constant and one observes a ballistic dispersion phase,

$$A^2(\tau) = 2E\tau^2 \tag{6}$$

where E is the mean kinetic energy of the advecting flow. A standard way of representing absolute dispersion is to define a time-dependent dispersion coefficient, $K(\tau)$, as

$$K(\tau) = \frac{A^2(\tau)}{2\tau} \quad . \tag{7}$$

In the ballistic phase, $K(\tau) \to E\tau$ as $\tau \to 0$, while in the Brownian dispersion phase $K(\tau) \to K$ for $\tau \to \infty$. The ballistic regime is sometimes visible in the dispersion curves computed from surface drifter data [43]. Subsurface float trajectories are often characterized by a well-defined ballistic regime, associated with very steep Lagrangian spectra at small times [44].

Lagrangian stochastic models (LSM) are employed to reproduce the main statistical properties of particle trajectories in turbulent flows, without resolving the full Eulerian dynamics. Individual trajectories computed by an LSM usually do not have the same characteristics of the particles advected by a realistic flow. The similarity is recovered—if ever!—only statistically, after averaging over particle ensembles and over different realizations of the turbulent flow. Thus, one should not expect an individual stochastic trajectory to resemble an individual float trajectory.

One simple class of stochastic models describes the process of single-particle dispersion. In this case, the spatial correlations of the advecting flow are discarded insofar as they do not translate into temporal correlations of the Lagrangian velocities (see also Rupolo et al. [44] for a discussion of how Eulerian spatial correlations are related to Lagrangian time correlations). A more complex approach deals with particle separation processes, i.e., relative dispersion. In this case, the stochastic model describes the time evolution of the separation of a particle pair, and spatial correlations of the turbulent flow become an essential ingredient of the picture. In the following, we shall consider only single-particle dispersion and the related stochastic descriptions. An exhaustive discussion of the atmospheric applications of Lagrangian stochastic models can be found in the monograph by Rodean [45]; for oceanographic applications see Griffa [46] and Brickman and Smith [47].

The simplest stochastic model for single-particle dispersion is the random walk (or Markoff-0 model). In this approach, the particle displacements are randomly extracted from a Gaussian distribution, and there is no temporal correlation between subsequent displacements. If we assume that there is no mean flow advecting the particles and that the turbulent flow is statistically isotropic, we can write a Lagrangian stochastic differential equation for the random walk as

$$\mathrm{d}\mathbf{X}_i = \sqrt{K}\mathrm{d}\mathcal{W}_i(t). \tag{8}$$

where $\mathbf{X}$ is the position of the ith particle and the diffusivity, K, is not allowed to vary in space and time. The incremental Wiener random vector, $\mathrm{d}\mathcal{W}_i$, has zero mean and it is δ-correlated in space and time, $\langle \mathrm{d}\mathcal{W}_i(t) \cdot \mathrm{d}\mathcal{W}_j(t') \rangle = \delta_{ij}\, \delta(t - t')\mathrm{d}t$.

The single-particle stochastic description illustrated above can be framed in terms of a deterministic partial differential equation for the time evolution of the probability density function of particle positions, $P(\mathbf{X}|\mathbf{X}(0), t)$. Defining the particle concentration at $\mathbf{x}$ as $\rho(\mathbf{x}, t) = \int P(\mathbf{X} = \mathbf{x}|\mathbf{X}(0), t)\ d\mathbf{X}(0)$, the

Fokker–Planck equation for the evolution of P gives the well-known diffusion equation:

$$\frac{\partial \rho(\mathbf{x},t)}{\partial t} = \frac{1}{2} K \nabla^2 [\rho(\mathbf{x},t)]. \tag{9}$$

The assumption of uncorrelated displacements is equivalent to the assumption that the Eulerian fluid velocities decorrelate instantaneously, i.e., that the turbulent structure of the flow has no correlations. In general, this assumption is not appropriate for ocean mesoscale flows, where the temporal correlations of the advecting velocity field cannot be discarded. The simplest way of accounting for a memory in Lagrangian velocities is to consider a Markoff-1 model. In this approach, the time evolution of the Lagrangian velocity of the ith particle, $\mathbf{U}_i$, is described by an Ornstein–Uhlenbeck (OU) process:

$$\mathrm{d}\mathbf{U}_i = -\frac{\mathbf{U}_i}{T}\mathrm{d}t + \sqrt{\frac{2\sigma^2}{T}}\mathrm{d}\mathcal{W}_i. \tag{10}$$

where T is the Lagrangian correlation time and σ^2 is the variance of the Lagrangian velocities. The first term on the r.h.s. is the (deterministic) fading-memory term, and the second term is the "stochastic kick," or random component, of the velocity fluctuation. For this process, the velocity distribution is a Gaussian with zero mean and variance σ^2, and the velocity autocorrelation is an exponential, $R(\tau) = \exp(-\tau/T)$. The (time-dependent) diffusion coefficient can be computed analytically,

$$K(\tau) = \sigma^2 T \left[1 - \frac{T(1 - e^{-\tau/T})}{t}\right], \tag{11}$$

see Griffa [46] for a discussion of this type of stochastic model in the context of oceanographic applications.

In a study of particle dispersion in two-dimensional turbulence, Pasquero et al. [21] showed that the linear Ornstein–Uhlenbeck model provides a good representation of absolute dispersion at short and large times (respectively in the ballistic and Brownian regimes), while at intermediate times it provides estimates of the dispersion coefficient which differ by at most 25% from the values obtained by direct integration of particle dynamics in the turbulent flow. If this discrepancy is acceptable, due for example to uncertain or poorly resolved data, then the use of the Ornstein–Uhlenbeck model is sufficient. To obtain a more precise estimate of the dispersion coefficient, however, a stochastic model that more closely represents the processes of particle dispersion in vortex-dominated mesoscale turbulence is warranted.

Major differences between the Ornstein–Uhlenbeck process and particle dispersion in mesoscale turbulence are related to the facts that the velocity distribution is non-Gaussian [20], the velocity autocorrelation is non-exponential [21], and particles get trapped in vortices for long times [39, 40]. Given these differences, it is indeed surprising that just a 25% discrepancy between the turbulent and the modeled dispersion coefficient is detected.

In an attempt to improve stochastic parameterizations of particle dispersion in mesoscale ocean turbulence, various extensions of the Ornstein–Uhlenbeck model have been proposed. The indications that Lagrangian accelerations in the ocean are correlated in time [44] have stimulated the development of Markoff-2 models where an Ornstein–Uhlenbeck formulation is written for the acceleration $\mathbf{a}$, with $\mathrm{d}\mathbf{U} = \mathbf{a}\ \mathrm{d}t$ [46]. Higher order models have also been proposed, with the aim of better reproducing other statistical properties of Lagrangian motions such as the sub- or super-diffusive behavior at intermediate times [48]. Superdiffusion has also been obtained by Reynolds [49], using a variation of a Markoff-2 model that includes spin.

Models that include spin have been designed to explicitly describe particle motion in and around coherent structures. In the presence of coherent vortices, particle motion has a rotational component, as evident in the looping trajectories of floats deployed inside mesoscale eddies. The rotational component of the velocity vector along a Lagrangian trajectory is characterized by an acceleration orthogonal to the trajectory. Simple geometrical arguments show that the introduction of the spin in Markoff-1 models corresponds to adding a new term in the stochastic equation for the velocity increment, proportional to the orthogonal velocity component [49, 50]. The individual trajectories produced by these models display spiraling motion, although the ensemble averaged velocity autocorrelation function is not necessarily oscillatory [49]. This model has recently been used to reproduce some statistical properties of Northwest Atlantic float trajectories [51].

On the other hand, it is not clear whether particle spinning inside vortices has any effect on space and timescales larger than those of the vortices themselves. In general, rotational motion inside vortices does not contribute to the large-scale spreading of particles; it is only the motion of the vortex itself that is responsible for particle displacements at large scales. In turn, vortices move because they are advected by other vortices and there is no self-induction of the vortices themselves [33]. As a result, the large-time dispersion properties of Lagrangian particles inside or outside the vortices of two-dimensional turbulence are the same. Thus, for the purpose of understanding particle dispersion at scales larger than the size of the individual vortices, the parameterization of particle motion inside a vortex can probably be neglected. Note, however, that the situation can be very different if the scale of motion of interest are large enough that variations with latitude of the Coriolis parameter, equal to twice the component of the Earth's angular velocity, cannot be neglected [52]. Vortices, indeed, move differently with respect to fluid particles in the background turbulence in presence of differential rotation. Here, significant differences between long-time dispersion properties of particles inside and outside vortices can be detected [52].

In a study of single-particle dispersion in two-dimensional turbulence, Pasquero et al. [21] proposed a parameterization of dispersion in two-dimensional turbulence at scales larger than those of the individual vortices. In doing so, no a priori difference between particles inside and outside vortices is drawn.

The main point of the approach followed in [21] is the observation that the Eulerian velocity at any point is determined by the combined effect of the far field of the vortices and the contribution of the local vorticity field in the background [20]. Thus, even outside vortices, the velocity field induced by the coherent vortices cannot be discarded: on average, 80% of the kinetic energy in the background turbulence outside vortices is due to the velocity field induced by the vortex population. In addition, the non-Gaussian velocities measured in the background turbulence outside vortices are entirely due to the action of the surrounding vortices, which extend their influence far away from their inner cores. This is a signature of the non-locality of the velocity field: a particle moving in a vortex-dominated flow is heavily affected by the vortex dynamics even if it is not located inside them.

In this approach, the stochastic Lagrangian velocity of a particle at the position $\mathbf{X}(t)$ is produced by the sum of two components,

$$\mathbf{U}(\mathbf{X}) = \mathbf{U}_{\mathrm{B}}(\mathbf{X}) + \mathbf{U}_{\mathrm{V}}(\mathbf{X}) \ , \tag{12}$$

where $\mathbf{U}_{\mathrm{B}}(\mathbf{X})$ is the velocity induced by the background turbulence and $\mathbf{U}_{\mathrm{V}}(\mathbf{X})$ is that induced by the vortices. The background-induced velocity is characterized by small energy and slow dynamics (i.e., long temporal correlations), while the vortex-induced component has large energy and it undergoes fast dynamics (whose temporal scale is of the order of the eddy turnover time). In addition, the vortex-induced component is characterized by a non-Gaussian velocity PDF.

A different stochastic equation has then to be used for each of the two components. Since the background-induced velocity component, $\mathbf{U}_{\mathrm{B}}(\mathbf{X})$, has a Gaussian distribution, a standard stochastic OU process can be used to describe it. As for the non-Gaussian, vortex-induced component $\mathbf{U}_{\mathrm{V}}(\mathbf{X})$, a proper description is easily obtained by considering a non-linear Markoff-1 model [21]. In this case, one needs to consider a generalized Langevin equation

$$\mathrm{d}\mathbf{U}_V = \mathbf{a}(\mathbf{U}_{\mathrm{V}})\mathrm{d}t + b(\mathbf{U}_{\mathrm{V}})\mathrm{d}\mathcal{W} \tag{13}$$

where the functions $\mathbf{a}$ and b are functions of the velocity $\mathbf{U}_{\mathrm{V}}$. The choice of the function $\mathbf{a}(\mathbf{U}_{\mathrm{V}})$ is (not uniquely) determined by the corresponding Fokker–Planck equation, with the use of the well-mixed condition [53]. In the end, the model proposed by Pasquero et al. becomes (we omit the particle index i for simplicity of notation):

$$\begin{aligned} \mathrm{d}\mathbf{X} &= (\mathbf{U}_{\mathrm{B}} + \mathbf{U}_{\mathrm{V}})\ \mathrm{d}t \\ \mathrm{d}\mathbf{U}_{\mathrm{B}} &= -\frac{\mathbf{U}_{\mathrm{B}}}{T_{\mathrm{B}}}\mathrm{d}t + \sqrt{\frac{2\sigma_{\mathrm{B}}^2}{T_{\mathrm{B}}}}\mathrm{d}\mathcal{W}_{\mathrm{B}} \\ \mathrm{d}\mathbf{U}_V &= -\frac{2 + |\mathbf{U}_{\mathrm{V}}|/\sigma_{\mathrm{V}}}{(1 + |\mathbf{U}_{\mathrm{V}}|/\sigma_{\mathrm{V}})^2}\ \frac{\mathbf{U}_{\mathrm{V}}}{T_{\mathrm{V}}}\ \mathrm{d}t + \sqrt{\frac{2\sigma_{\mathrm{V}}^2}{T_{\mathrm{V}}}}\mathrm{d}\mathcal{W}_{\mathrm{V}} \end{aligned} \tag{14}$$

where $T_B > T_V$, $\sigma_V^2 \gg \sigma_B^2$, and $\mathcal{W}_B$ and $\mathcal{W}_V$ are two independent Wiener processes.

Interestingly, the parameters of the stochastic model depicted above can be obtained from fits to an ensemble of Lagrangian trajectories (i.e., assuming no knowledge of the advecting velocity field). Comparison with particle advection in two-dimensional turbulence shows that this model captures single-particle dispersion with an error of less than 5%, and it does also capture statistical quantities measuring higher-order moments of the dispersion statistics (e.g., the distribution of first-exit times). Note that both the non-linear nature of the vortex-induced velocity and the presence of a low-energy background-induced velocity are essential ingredients of the model. At shorter times, the vortex-induced velocity dominates and it entirely determines statistical properties such as the non-Gaussian velocity distribution. At longer times, the vortex-induced velocity becomes rapidly uncorrelated and the lower-energy background-induced velocity gives a significant contribution to particle dispersion.

One advantage of the model illustrated above is that it has been built from a detailed knowledge of the dynamics of vortex-dominated flows. That is, it is not obtained by ignoring the structure of the flow, but from an attempt to reproduce, in a stochastic framework, some of the essential ingredients of mesoscale turbulence. In particular, this model fully exploits the two-component nature of mesoscale turbulence.

4 Dynamics of Passive and Active Tracers

Transport processes can be approached from an Eulerian perspective, focusing on the advection–diffusion equation for an advected tracer field concentration:

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = F_\rho + D_\rho$$

where ρ is the concentration of the advected tracer, and F_ρ and D_ρ are respectively source and sink terms for the tracer.

There is a deep difference between the dynamics of active and passive tracers. A passive tracer does not feed back on the velocity field, as in the case of the concentration of a (dispersed) pollutant or of plankton in the ocean. An active tracer, on the other hand, does feed back on the fluid dynamics, think of temperature in a convecting fluid or vorticity in two-dimensional turbulence (which indeed defines the velocity field by the Biot-Savart law $\omega = \nabla^2\psi$ and $u = -\partial\psi/\partial y$, $v = \partial\psi/\partial x$). Clearly, to some extent all tracers are "active," either dynamically or thermodynamically. However, it is often assumed that when the feedback is small, or indirect, it can be discarded.

In the absence of sources and sinks, both the spatial average, $\langle\rho\rangle$, and the variance, $\langle(\rho - \langle\rho\rangle)^2\rangle$, of the tracer concentration are conserved. Without loss of generality, we can put $\langle \rho \rangle = 0$. In a statistically stationary flow, the dynamics of a passive tracer in two-dimensional turbulence is characterized by a direct cascade of tracer variance from large to small scales. In the inertial range, far from the characteristic scales of sources and sinks, dimensional arguments indicate that the tracer variance spectrum, $P_\rho(k)$ where k is the wavenumber, is characterized by a form $P_\rho(k) \propto k^{-1}$ [54].

The situation for vorticity is complicated by the presence of two quadratic invariants when $F = D = 0$: enstrophy, $Z = \langle \omega^2 \rangle$ (again we have made the safe assumption that $\langle \omega \rangle = 0$), analogous to tracer variance, and energy, $E = \langle (u^2 + v^2) \rangle / 2$. The simultaneous conservation of these two quantities induces a direct cascade of enstrophy, analogous to the direct cascade of tracer variance, and an inverse cascade of energy, a specific property of two-dimensional turbulence [55, 56]. In the case that the scales of small-scale dissipation, l_D, of forcing, l_F, and of large-scale boundary effects, L, are sufficiently far from each other, dimensional arguments can be used to determine the form of the spectrum. The inverse energy cascade takes place at scales l that are larger than the forcing scale, $l_\mathrm{F} < l < L$, and it is associated with an energy spectrum $E(k) \propto k^{-5/3}$. At scales smaller than the forcing scales, $l_\mathrm{D} < l < l_\mathrm{F}$, a direct cascade of enstrophy appears, associated with an energy spectrum $E(k) \propto k^{-3}$ and an enstrophy spectrum $Z(k) \propto k^{-1}$.

Direct numerical simulation of forced-dissipated two-dimensional turbulence indicates that the spectrum of passive tracer variance follows with a good approximation the predicted scaling form $P_\rho \propto k^{-1}$. On the other hand, the enstrophy spectrum in the range of the direct enstrophy cascade is usually steeper than the prediction from dimensional arguments.

This difference has recently been explored by Babiano and Provenzale [57], who investigated why the direct cascade is weaker for vorticity than for a passive tracer. The analysis of the vorticity field by means of the local value of the Okubo–Weiss parameter [58, 59], $Q = s^2 - \omega^2$, where $s^2 = (\partial u/\partial x - \partial v/\partial y)^2 + (\partial u/\partial y + \partial v/\partial x)^2$, has shown that the enstrophy cascade is reversed in elliptic regions characterized by dominance of rotation over strain ($Q < 0$). In the cores of the vortices and in small elliptic patches in the background, at finite scales in the enstrophy inertial range one observes an *inverse* enstrophy cascade. In turn, this is associated with gradient-smoothing processes and an inverse energy cascade.

This behavior is consistent with the weaker spectral enstrophy flux, compared to the passive tracer variance flux, and with the steeper logarithmic slope of the enstrophy spectrum. The inversion of the enstrophy cascade in elliptic regions is the main difference between the dynamics of passive tracer and vorticity. In particular, Babiano and Provenzale speculated that the inversion of the enstrophy cascade can be one important mechanism associated with the formation of coherent vortices.

5 Conclusions

Geophysical turbulence is populated with long-lived, energetic structures: vortices, fronts, jets, and waves. Among these, coherent vortices play an especially important role, and affect transport processes in many ways.

As a consequence, transport processes cannot be understood in detail by resorting to simple stochastic parameterizations, but require the development and use of new approaches. In this chapter we have discussed some possible options, that include non-linear stochastic processes and an explicit consideration of the turbulent cascades.

Of course, many issues are still open. One conceptual question is how and why do coherent vortices form. The consideration of the cascades can help address this problem, but much more needs to be done.

Another active topic of research, which has not been discussed here, is the interplay of coherent vortices and the marine ecosystem (see the contribution of Marina Levy in this volume, or Pasquero et al. [60, 61] to discover the view of some of the authors of the present chapter). Mesoscale vortices affect the population dynamics of phyto- and zooplankton, and are associated with secondary currents responsible for localized vertical fluxes of nutrients [62, 63, 64, 65, 66, 67, 68, 69, 70]. The fact that the nutrient fluxes have a fine spatial and temporal detail, generated by the eddy field, has important consequences on primary productivity [60, 65, 71]. Furthermore, vortices can act as shelters for temporarily less-favored planktonic species owing to their trapping properties [72] and can disguise the possible presence of self-sustained oscillations in the plankton system [73]. The horizontal velocity field induced by vortices also plays an important role in determining plankton patchiness [74, 75, 76]. The parameterization of transport in mesoscale turbulence and of its ecological effects [77], needed for properly representing biogeochemical cycles in coarse-resolution climate models, is a key open problem.

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