1. The choice of vertical coordinate turns out to be very important in simplifying dynamic equations. Two typical coordinates are height and pressure, but we will see that potential temperature is a very convenient coordinate.

Show that the horizontal pressure gradient force in height coordinates can be rewritten in isentropic coordinates as:

\[
\frac{1}{\rho} \left( \frac{\partial p}{\partial x} \right)_z = \left( \frac{\partial M}{\partial x} \right)_\rho
\]

where \( M = c_p T + \Phi \) is the Montgomery streamfunction. The subscripts indicate coordinate system along which horizontal is defined.

**Bonus question:** Why are potential temperature coordinates called isentropic?

2. The first law of thermodynamics expresses conservation of energy.

\[
c_p \frac{d \ln T}{dt} - R \frac{d \ln p}{dt} = \frac{J}{T}
\]

Starting here, do a scale analysis to show that an approximate form tells us that temperature changes are mostly due to imbalance between temperature advection and work done by expansion or compression (i.e., vertical transport of deviation from the dry adiabatic lapse rate):

\[
\frac{\partial T}{\partial t} \approx - \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) + w(\Gamma - \Gamma_d)
\]

(Hint: Recall, Holton pg. 53, we can express a quantity as the some of a reference state and some deviation. E.g., \( T_{total}(x,y,z,t) = T_0(z) + T(x,y,z,t) \), which leads to a useful expansion of \( \ln T_{total} \))

3. Read Hadley’s (short) paper from 1735 (available on the class web site). What physical principle was he evoking to explain the easterlies in the tropics? What was, as he explains, the problem with his explanation?