Momentum

Atmosphere conserves

- Energy (mechanical, heat, …)
- Mass (of air, and other gases)
- Momentum (and angular momentum)

To describe motion, we can make use of:

- Newton's 2nd law ($\sum F = ma$)
- First Law of Thermodynamics
- We need to account for fact that earth is rotating, spherical and there is gravity
Some assumptions

- Atmosphere is thin so distance from center is about the same as the earth's radius.
- Gravity is constant
- Given these, also neglect horizontal component of rotation ensures momentum equations conserve momentum.

*These are called the “traditional approximation”*

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**Primitive equations**

*basic building blocks*

- Momentum (horizontal)
  \[
  \frac{du}{dt} = fv - \frac{1}{\rho} \frac{\partial p}{\partial x}
  \]
  \[
  \frac{dv}{dt} = -fv - \frac{1}{\rho} \frac{\partial p}{\partial y}
  \]
  \[
  \frac{\partial p}{\partial z} = -\rho g
  \]

- Hydrostatic (vertical)
  \[
  \frac{1}{\rho} \frac{dp}{dt} = -\left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)
  \]

- Continuity
  \[
  \frac{d\rho}{dt} = \frac{1}{c_p} \left( \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) - \alpha \frac{dp}{dt} \right)
  \]

- Thermodynamic
  \[
  \frac{dT}{dt} - \alpha \frac{dp}{dt} = J
  \]

- (and equation of state: ideal gas)
  \[
  p = \rho RT
  \]

Boundary layers add an extra force for momentum equations, and contributes to $J$ in thermodynamic equation

Otherwise, a closed set of PDEs See Holton Ch2 for full derivation
NCEP Reanalysis data

500 mb wind field and heights for 00Z15NOV1975

Westerly wind and temperature

Temperature and wind field related (thermal wind balance)
i.e., $\partial u / \partial p \sim \partial T / \partial y$

- Knowing temperature gradient, estimate jet.
- Knowing jet, estimate gradient.