## Atmospheric Thermodynamics

## Water Vapor in Air

Why do we care about water in the atmosphere?
We have already discussed vapor pressure (e) as one way to describe the amount of water vapor present in air, but meteorologists have several other measures of water vapor that they use.

Mixing ratio ( $w$ ): The mass of water vapor contained in a unit mass of dry air

$$
w \equiv \frac{m_{v}}{m_{d}} \text { (units: } \mathrm{kg} / \mathrm{kg} \text { ) }
$$

$m_{v}$ : mass of water vapor (units: kg )
$m_{d}$ : mass of dry air (units: kg )
Specific humidity (q): Mass of water vapor contained in a unit mass of air

$$
q \equiv \frac{m_{v}}{m_{d}+m_{v}}(\text { units: } \mathrm{kg} / \mathrm{kg})
$$

The following equations allow for conversion between mixing ratio and specific humidity:

$$
w=\frac{q}{1-q} \quad q=\frac{w}{1+w}
$$

The vapor pressure can be calculated from the mixing ratio by noting that the partial pressure exerted by any gas in a mixture of gases is proportional to the number of moles of that gas in the mixture.
$e=\frac{n_{v}}{n_{d}+n_{v}} p=\frac{\frac{m_{v}}{M_{w}}}{\frac{m_{d}}{M_{d}}+\frac{m_{v}}{M_{w}}} p$

Multiplying this expression by: $\frac{\frac{M_{w}}{m_{d}}}{\frac{M_{w}}{m_{d}}}$ gives:
$e=\frac{\frac{m_{v}}{m_{d}}}{\frac{M_{w}}{M_{d}}+\frac{m_{v}}{m_{d}}} p=\frac{w}{\varepsilon+w} p$, where $\varepsilon=\frac{M_{w}}{M_{d}}$
The mixing ratio or specific humidity can also be calculated from the vapor pressure using:

$$
w=\frac{\varepsilon e}{p-e} \approx \frac{\varepsilon e}{p}
$$

$$
q=\varepsilon \frac{e}{p-(1-\varepsilon) e}
$$

The virtual temperature can also be calculated in terms of mixing ratio, rather than vapor pressure, as:
$T_{v} \equiv \frac{T}{1-\frac{e}{p}(1-\varepsilon)}=\frac{T}{1-\left(\frac{w}{\varepsilon+w}\right)(1-\varepsilon)}=T \frac{\varepsilon+w}{\varepsilon(1+w)}$
Example: The pressure in a hurricane is observed to be 950 mb . At this time the temperature is 88 deg F and the vapor pressure is 25 mb .

Calculate the mixing ratio and specific humidity of the air in this hurricane.

The mixing ratio, specific humidity, and vapor pressure are all absolute measures of the amount of water vapor in the air. Each of these variables varies directly with the amount of water vapor present.

Consider a closed box with pure water at the bottom of the box.
Assuming that the air in the box initially contains no water vapor how will the amount of water vapor in the air change over time?

What impact will this change in the amount of water vapor have on the vapor pressure?

(a) Unsaturated

(b) Saturated

The air in the box is said to be unsaturated if the rate of condensation of water vapor is less than the rate of evaporation from the water surface.

Saturation: A dynamic equilibrium between air and a water surface in which there are as many water molecules returning to the water surface (condensing) as there are escaping (evaporating).

When air is saturated water vapor will condense to form liquid water.
Saturation vapor pressure ( $e_{s}$ ): The partial pressure that would be exerted by water vapor molecules in a given volume of the atmosphere if the air were saturated

The saturation vapor pressure depends only on temperature, and increases as temperature increases.

What is the physical explanation for this temperature dependence of the saturation vapor pressure?

The mathematical basis for this relationship between $T$ and $e_{s}$ will be explored later in this chapter.


We can consider an analogous situation with ice instead of liquid water in a closed box.

How will the saturation vapor pressure over an ice surface differ from that over a liquid surface?
$e_{s i}$ - saturation vapor pressure with respect to an ice surface
$e_{s}(T)>e_{s i}(T)$

Using the idea of saturation we can define additional saturation variables:
Saturation mixing ratio $\left(w_{s}\right)$ : The mass of water vapor contained in a unit mass of dry air if the air were saturated

$$
w_{s} \equiv \frac{m_{v s}}{m_{d}}=\frac{\varepsilon e_{s}}{p-e_{s}} \approx \frac{\varepsilon e_{s}}{p}
$$

$m_{v s}$ : mass of water vapor in air that is saturated with respect to a liquid water surface (units: kg) $m_{d}$ : mass of dry air (units: kg )

Saturation specific humidity $\left(q_{s}\right)$ : Mass of water vapor contained in a unit mass of air if the air were saturated
$q_{s}=\varepsilon \frac{e_{s}}{p-(1-\varepsilon) e_{s}}$
The "saturation" humidity variables (saturation vapor pressure, saturation mixing ratio, and saturation specific humidity) all depend on temperature.


Information plotted on this skew T $-\log p$ diagram is the same as for a dry skew T - $\log p$ plus:

Saturation mixing ratio: Blue dashed lines (Units: $\mathrm{g} \mathrm{kg}^{-1}$ ) (Also used to indicate mixing ratio)

How does $w_{s}$ vary with temperature at constant pressure? How does $w_{s}$ vary with pressure at constant temperature?

What condition describes saturated air?
What two changes can occur to cause air to become saturated in the atmosphere?

For an air parcel that contains a non-zero amount of water vapor the air parcel can be cooled sufficiently such that it eventually becomes saturated. The temperature to which this air parcel must be cooled, at constant pressure, is known as the dew point temperature.

Dew point temperature ( $T_{d}$ ): The temperature to which an air parcel must be cooled at constant pressure in order for the air parcel to become saturated with respect to a plane surface of pure water.

Dew point temperature is the most commonly reported humidity variable on weather maps.

Remember: Temperature always determines the amount of moisture required for air to become saturated, while dew point temperature always indicates the ACTUAL amount of water vapor in the air.

Relative humidity $(R H)$ : The ratio of the mixing ratio to the saturation mixing ratio - an indication of how close the air is to being saturated

$$
R H \equiv \frac{w}{w_{s}} \times 100 \% \approx \frac{e}{e_{s}} \times 100 \%(\text { units: \%) }
$$

Relative humidity will be high when $w \approx w_{s}, e \approx e_{s}, q \approx q_{s}$, or $T_{d} \approx T$.
Relative humidity will be low when $w \ll w_{s}, e \ll e_{s}, q \ll q_{s}$, or $T_{d} \ll T$.
Relative versus absolute measures of humidity


Example: Skew T diagrams and moisture variables using ATOC weather station observations

Determine the mixing ratio and saturation mixing ratio at SEEC using the skew T diagram and the reported temperature, dew point temperature, and pressure.

How does the relative humidity calculated from these values compare to the reported relative humidity?

Lifting condensation level (LCL) - the level to which an unsaturated (but moist) parcel of air must be lifted adiabatically in order to just become saturated with respect to a plane surface of pure water

How does the potential temperature and mixing ratio of an air parcel change as it is lifted adiabatically?

Example: Determine the LCL for an air parcel lifted adiabatically from the ATOC weather station.


## Latent heats

During a phase change of a system (e.g. ice to liquid water) the molecular configuration of the molecules in the system is altered, resulting in a change in the system's internal energy. This change in internal energy can result without a change in the molecular kinetic energy (and thus temperature) of the system.

The heat associated with this change in internal energy is referred to as the latent heat.

Latent heat of melting $\left(L_{m}\right)$ - heat given to a unit mass of material to convert it from the solid to the liquid phase without a change in temperature

Latent heat of freezing - heat released from a unit mass of material when the material is converted from the liquid to the solid phase without a change in temperature
$L_{m}=3.34 \times 10^{5} \mathrm{~J} \mathrm{~kg}^{-1}$ for water at 1013 hPa and a temperature of 0 deg C
The latent heat of freezing has the same value as the latent heat of melting.
Latent heat of vaporization $\left(L_{v}\right)$ - heat given to a unit mass of material to convert it from the liquid to the vapor phase without a change in temperature

Latent heat of condensation - heat released from a unit mass of material when the material is converted from the vapor to the liquid phase without a change in temperature
$L_{v}=2.25 \times 10^{6} \mathrm{~J} \mathrm{~kg}^{-1}$ for water at 1013 hPa and 100 deg C
The latent heat of condensation has the same value as the latent heat of vaporization.

When liquid water in the atmosphere evaporates where does the latent heat for this phase change come from?

When water vapor condenses in the atmosphere where does the latent heat from this phase change go?

Wet bulb temperature ( $T_{w}$ ): The temperature to which air may be cooled by evaporating water into it at constant pressure until saturation is reached.

How does the wet bulb temperature differ from the dew point temperature?
For a sample of moist air undergoing an isobaric process the first law of thermodynamics gives:
$d q=c_{p} d T$,
where we have neglected the mass of water in the sample.
Evaporation of mass of water $d w$ requires an addition of heat $(d q)$ given by:
$d q=-L_{v} d w$
These equations give:
$c_{p} d T=-L_{v} d w$
which describes the change in temperature when mass of water $d w$ evaporates in the wet bulb process.

This equation can be integrated to give:

$$
\begin{aligned}
& \int_{T_{w}}^{T} d T=-\frac{L_{v}}{c_{p}} \int_{w_{s}}^{w} d w \\
& T-T_{w}=-\frac{L_{v}}{c_{p}}\left(w-w_{s}\right) \\
& \frac{T-T_{w}}{w_{s}-w}=\frac{L_{v}}{c_{p}}
\end{aligned}
$$

This equation can then be solved for the wet-bulb temperature, $T_{w}$ :
$T_{w}=T-\frac{L_{v}}{c_{p}}\left(w_{s}-w\right)$

## Saturated Adiabatic and Pseudoadiabatic Processes

Does the temperature of an air parcel continue to decrease at the dry adiabatic rate when lifted beyond the lifting condensation level?

Once an air parcel is lifted beyond the lifting condensation level latent heat is released as water vapor condenses and the potential temperature of the air parcel increases.

If the latent heat released as water vapor condenses remains in the rising air parcel and all of the condensate remains in the air parcel this process may still be considered adiabatic (and reversible) and is referred to as a saturated adiabatic process.

If the latent heat released as water vapor condenses remains in the air parcel but the condensate immediately falls out of the air parcel the process is no longer strictly adiabatic, since the condensate will carry some heat out of the parcel, and the process is referred to as a pseudoadiabatic process.

The amount of heat carried by the condensate is small compared to the heat carried by the air in the parcel so saturated adiabatic and pseudoadiabatic processes are virtually identical.

The changes in temperature, pressure, and saturation mixing ratio for a pseudoadiabatic process can be found by considering the first law of thermodynamics:
$\frac{d q}{T}=c_{p} \frac{d T}{T}-R \frac{d p}{p}$
Noting that for a saturated process as $T$ decreases $w_{s}$ will decrease and water vapor will condense, releasing latent heat. The latent heat (dq) released by this condensation is given by:

$$
d q=-L_{v} d w_{s}
$$

Substituting this into the first law of thermodynamics gives:
$-\frac{L_{v} d w_{s}}{T}=c_{p} \frac{d T}{T}-R \frac{d p}{p}$
This equation indicates the relationship between changes in temperature, pressure, and saturation mixing ratio that occur in a pseudoadiabatic process.

Moist adiabats (curved black lines that slope up and to the left on a skew T diagram) graphically represent this relationship.


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A saturated air parcel that is undergoing a moist adiabatic (or pseudoadiabatic) displacement will move parallel to the moist adiabats on a skew T diagram.

Example: Determine the temperature and dew point temperature of an air parcel lifted from the roof of Duane Physics to 200mb

Saturated adiabatic lapse rate $\left(\Gamma_{s}\right)$ - rate of decrease of temperature with height for an air parcel that is rising or sinking under saturated adiabatic conditions. This is also referred to as the moist adiabatic lapse rate.

How does the magnitude of $\Gamma_{d}$ compare to the magnitude of $\Gamma_{s}$ ?
Equivalent potential temperature $\left(\theta_{e}\right)$ - the potential temperature an air parcel would have if all of the water vapor in the parcel were condensed

To derive an expression for $\theta_{e}$ start with the first law of thermodynamics expressed as:
$-\frac{L_{v} d w_{s}}{T}=c_{p} \frac{d T}{T}-R \frac{d p}{p}$
Taking In of Poisson's equation gives:
$\ln \theta=\ln T-\frac{R}{c_{p}} \ln p+\frac{R}{c_{p}} \ln p_{0}$
Differentiation of this equation gives:
$\frac{d \theta}{\theta}=\frac{d T}{T}-\frac{R}{c_{p}} \frac{d p}{p}$
$c_{p} \frac{d \theta}{\theta}=c_{p} \frac{d T}{T}-R \frac{d p}{p}$

Substitution of this into the first law of thermodynamics gives:

$$
\begin{aligned}
& -\frac{L_{v} d w_{s}}{T}=c_{p} \frac{d \theta}{\theta} \\
& -\frac{L_{v}}{c_{p} T} d w_{s}=\frac{d \theta}{\theta}
\end{aligned}
$$

Noting that: $\frac{L_{v}}{c_{p} T} d w_{s} \approx d\left(\frac{L_{v} w_{s}}{c_{p} T}\right)$ gives:
$-d\left(\frac{L_{v} w_{s}}{c_{p} T}\right) \approx \frac{d \theta}{\theta}$
Integrating this equation from an initial state of $T, w_{s}$, and $\theta$ (the air parcel state at the LCL) to a final state in which all of the water vapor has been condensed (and $w_{s} / T \rightarrow 0$ and $\theta \rightarrow \theta_{e}$ ) gives:

$$
\begin{aligned}
& \int_{w_{s} / T}^{0}-d\left(\frac{L_{v} w_{s}}{c_{p} T}\right) \approx \int_{\theta}^{\theta_{e}} \frac{d \theta}{\theta} \\
& \frac{L_{v} w_{s}}{c_{p} T}=\ln \theta_{e}-\ln \theta=\ln \frac{\theta_{e}}{\theta}
\end{aligned}
$$

This can be solved for the equivalent potential temperature, $\theta_{e}$, to give:
$\theta_{e}=\theta \exp \left(\frac{L_{v} w_{s}}{c_{p} T}\right)$
Note: If the air parcel is initially unsaturated then the values of $w_{s}$ and $T$ used to calculate $\theta_{e}$ are the saturation mixing ratio and temperature of the air parcel at the lifting condensation level.

As will be shown below $\theta_{e}$ can be easily determined using a skew $T-\log \mathrm{p}$ diagram.

The wet bulb temperature ( $T_{w}$ ) can be estimated using a skew $T-\log p$ diagram by noting the temperature of an air parcel that is brought moist adiabatically from the lifting condensation level to the initial pressure.

Wet bulb potential temperature $\left(\theta_{w}\right)$ : The temperature an air parcel would have if brought down moist adiabatically from the lifting condensation level to a pressure of 1000 mb .

Both $\theta_{w}$ and $\theta_{e}$ are conserved for both dry adiabatic and saturated pseudoadiabatic processes.

Using a skew $\mathrm{T}-\log \mathrm{p}$ diagram one can relate $T, T_{d}, T_{w}, \theta, \theta_{e}$, and $\theta_{w}$ as shown below.


Adiabatic liquid water content $(\chi)$ : The amount of water condensed during a pseudoadiabatic process.

This is given by the change in $w_{s}$ between the lifting condensation level and the level of interest.

Example: Using a skew $\mathrm{T}-\log \mathrm{p}$ diagram determine the following quantities ( $w, w_{s}, \mathrm{RH}, \theta, \mathrm{LCL}, T_{w}, \theta_{w}, \theta_{e}$ ) based on the current weather observation from the ATOC weather station.


How do the values of $T, T_{d}$, and $T_{w}$ compare for an unsaturated air parcel?
For a saturated air parcel?

How do the thermodynamic properties of an air parcel change when the air parcel is lifted above its LCL and then descends dry adiabatically?

Example: Determine $T, T_{d}, w, w_{s}, \mathrm{RH}$, and $\theta$ for an air parcel that crosses the Rocky Mountains from Grand Junction, CO to Boulder, CO

You may assume that all condensate immediately precipitates from the air parcel above the LCL.


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## Static Stability

## Environmental and Parcel Temperatures

As discussed above the temperature of an air parcel that is displaced vertically can be determined using a skew T diagram if it is assumed that the air parcel experiences adiabatic changes during the displacement. The rate of decrease of the air parcel temperature with height is given by either the dry $\left(\Gamma_{d}\right)$ or saturated $\left(\Gamma_{s}\right)$ adiabatic lapse rate.

In addition to the air parcel temperature we need to consider the temperature of the environment surrounding the air parcel. The rate of decrease of the environmental temperature with height is given by the environmental lapse rate $(\Gamma)$.

One way to measure the environmental temperature is with a radiosonde.

## Example: Plotting radiosonde data on a skew T diagram (KFWD 5 June 200500 UTC)

| LEV | $\begin{gathered} \text { PRES } \\ \mathrm{mb} \end{gathered}$ | $\begin{gathered} \text { HGHT } \\ \mathrm{m} \end{gathered}$ | $\begin{gathered} \text { TEMP } \\ \mathrm{C} \end{gathered}$ | $\begin{gathered} \text { DEWP } \\ \text { C } \end{gathered}$ | $\begin{array}{r} \mathrm{RH} \\ \% \\ \hline \end{array}$ | $\begin{array}{r} \text { DD } \\ \mathrm{C} \end{array}$ | $\begin{aligned} & \text { WETB } \\ & \text { C } \end{aligned}$ | $\begin{aligned} & \text { DIR } \\ & \text { deg } \end{aligned}$ | $\begin{aligned} & \text { SPD } \\ & \text { knt } \end{aligned}$ | $\begin{gathered} \text { THETA } \\ \mathrm{K} \end{gathered}$ | $\begin{gathered} \text { THE-V } \\ \mathrm{K} \end{gathered}$ | $\begin{gathered} \text { THE-W } \\ \mathrm{K} \end{gathered}$ | $\begin{gathered} \text { THE-E } \\ \mathrm{K} \end{gathered}$ | $\begin{aligned} & \mathrm{W} \\ & \mathrm{~g} / \mathrm{kg} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SFC | 982 | 196 | 31.4 | 22.4 | 59 | 9.0 | 24.7 | 165 | 10 | 306.1 | 309.4 | 298.5 | 359.1 | 17.63 |
| 2 | 925 | 728 | 26.0 | 20.0 | 70 | 6.0 | 21.7 | 175 | 21 | 305.9 | 308.9 | 297.5 | 354.2 | 16.11 |
| 4 | 850 | 1468 | 20.4 | 15.5 | 73 | 4.9 | 17.1 | 195 | 20 | 307.5 | 310.0 | 296.0 | 347.2 | 13.14 |
| 6 | 830 | 1675 | 23.2 | 5.2 | 31 | 18.0 | 12.4 | 203 | 21 | 312.6 | 313.8 | 292.6 | 333.4 | 6.69 |
| 7 | 771 | 2310 | 17.6 | 2.6 | 37 | 15.0 | 9.0 | 207 | 23 | 313.2 | 314.3 | 292.3 | 332.0 | 5.99 |
| 8 | 749 | 2557 | 16.0 | -6.0 | 21 | 22.0 | 5.2 | 201 | 23 | 314.1 | 314.7 | 290.2 | 324.6 | 3.26 |
| 9 | 700 | 3127 | 11.8 | -12.2 | 17 | 24.0 | 1.5 | 200 | 20 | 315.6 | 316.0 | 289.5 | 322.6 | 2.14 |
| 10 | 617 | 4170 | 4.6 | -13.4 | 26 | 18.0 | -2.8 | 247 | 28 | 318.9 | 319.3 | 290.6 | 326.2 | 2.20 |
| 11 | 544 | 5180 | -4.5 | -17.5 | 35 | 13.0 | -9.0 | 260 | 34 | 319.7 | 320.1 | 290.5 | 325.8 | 1.78 |
| 12 | 523 | 5490 | -6.3 | -31.3 | 12 | 25.0 | -12.3 | 253 | 29 | 321.2 | 321.3 | 289.7 | 323.1 | 0.53 |
| 13 | 500 | 5840 | -9.1 | -32.1 | 14 | 23.0 | -14.3 | 245 | 22 | 321.9 | 322.0 | 289.9 | 323.8 | 0.52 |
| 14 | 475 | 6232 | -12.3 | -33.3 | 16 | 21.0 | -16.7 | 235 | 25 | 322.7 | 322.8 | 290.1 | 324.5 | 0.49 |
| 16 | 400 | 7520 | -20.1 | -44.1 | 10 | 24.0 | -23.4 | 235 | 39 | 328.9 | 328.9 | 291.6 | 329.6 | 0.19 |
| 17 | 300 | 9590 | -35.5 | -57.5 | 9 | 22.0 | -36.8 | 235 | 41 | 335.3 | 335.3 | 293.2 | 335.6 | 0.05 |
| 18 | 250 | 10830 | -45.5 | -63.5 | 11 | 18.0 | -46.1 | 245 | 43 | 338.4 | 338.4 | 294.0 | 338.5 | 0.03 |
| 20 | 200 | 12280 | -54.1 | -69.1 | 14 | 15.0 | -54.4 | 240 | 54 | 347.1 | 347.1 | 296.0 | 347.2 | 0.02 |
| 23 | 150 | 14100 | -59.3 | -75.3 | 11 | 16.0 | -59.5 | 245 | 38 | 367.9 | 367.9 | 300.2 | 368.0 | 0.01 |
| 24 | 130 | 14990 | -60.5 | -77.5 |  | 17.0 | -60.7 | 245 | 50 | 381.1 | 381.1 | 302.4 | 381.2 | 0.01 |
| 25 | 100 | 16590 | -68.1 | -84.1 | 8 | 16.0 | -68.2 | 250 | 42 | 396.2 | 396.2 | 304.7 | 396.2 | 0.00 |

Archived radiosonde data like this can be found at:
http://vortex.plymouth.edu/uacalplt-u.html

Example: KFWD 5 June 200500 UTC


Example: Consider an air parcel lifted adiabatically from the surface to 200 mb , with the same initial temperature and dew point temperature as the environment.

Compare the temperature of this air parcel to the temperature of the environment at (a) 900 mb , (b) 800 mb , and (c) 500 mb ?

Static Stability in Unsaturated Air
Consider the situation shown below where $\Gamma<\Gamma_{\mathrm{d}}$


How will the temperature of the air parcel change if it is lifted from point O ?

How does the temperature of the air parcel at this new location compare to the temperature of the environment?

What does this difference in temperature imply about differences in density of the air parcel and environmental air?

How would this analysis change if the air parcel were forced to sink from point O?

An air parcel that is less dense than its environment will rise due to a buoyant force, while an air parcel that is more dense than its environment will sink due to a buoyant force.

What is the direction of the buoyant force acting on the air parcel in the example above if it is displaced up (down) from its original position at O ?

Consider an air parcel with volume $V$ at height $z$. Assume that this air parcel has the same temperature as its environment $\left(=T_{z}\right)$ and same volume as the environmental air it displaces.

At height $z+\delta z$ the air parcel has density $\rho_{p a r c e l, z+\delta z}$ and the displaced environmental air has density $\rho_{z+\delta z}$.

The buoyant force (per unit mass) acting on the air parcel is given by:

$$
\begin{aligned}
F_{B} & =\frac{\left(\begin{array}{cc}
\text { weight } & \text { of } \\
\text { air } & \text { displaced }
\end{array}\right)-\left(\begin{array}{cc}
\text { weight } & \text { of } \\
\text { air } & \text { parcel }
\end{array}\right)}{(\text { mass of parcel })} \\
& =\frac{g \rho_{z+d z} V-g \rho_{\text {parcel }, z+d z} V}{\rho_{\text {parcel }, z+d z} V} \\
& =g\left(\frac{\rho_{z+d z}-\rho_{\text {parcel }, z+d z}}{\rho_{\text {parcel }, z+d z}}\right)
\end{aligned}
$$

We can rewrite the expression for this force using the ideal gas law.

$$
\begin{aligned}
F_{B} & =g \frac{\left(\frac{p}{R_{d} T_{z+d z}}\right)-\left(\frac{p}{R_{d} T_{\text {parcel }, z+d z}}\right)}{\left(\frac{p}{R_{d} T_{\text {parcel }, z+d z}}\right)} \\
& =g \frac{\frac{1}{T_{z+d z}}-\frac{1}{T_{p a r c e l, z+d z}}}{\frac{1}{T_{p a r c e l, z+d z}}} \\
& =g \frac{T_{\text {parcel }, z+d z}-T_{z+d z}}{T_{z+d z}}
\end{aligned}
$$

The air parcel will experience an upward directed buoyant force when it is warmer than its environment and a downward directed buoyant force when it is cooler than its environment.

How does the temperature of an air parcel that is displaced upward compare to the temperature of its environment when:

$$
\Gamma<\Gamma_{d} ?
$$



Temperature
(a)

$$
\Gamma>\Gamma_{d} ?
$$


(b)
$\Gamma=\Gamma d$ ?
What does this imply about the direction of the buoyant force acting on this air parcel?

Static stability classes: Stable, unstable, and neutral


If the lapse rate of the environment $(\Gamma)$ is greater than $\Gamma_{d}$ then the environment is statically unstable, and an unsaturated air parcel that is initially displaced upward will continue to accelerate upward.

We can also consider the buoyant force as a function of potential temperature.

For both the air parcel and the environment at height $z$ :

$$
T_{z}=\theta_{z}\left(\frac{p_{z}}{p_{0}}\right)^{R_{d} / c_{p}}
$$

How does the temperature and potential temperature of the air parcel change as it is lifted adiabatically from height $z$ to height $z+d z$ ?

$$
T_{\text {parcel }, z+d z}=\theta_{z}\left(\frac{p_{z+d z}}{p_{0}}\right)^{R_{d} / c_{p}}
$$

The temperature of the environment at height $z+d z$ is:

$$
T_{z+d z}=\theta_{z+d z}\left(\frac{p_{z+d z}}{p_{0}}\right)^{R_{d} / c_{p}}
$$

Using this the buoyant force can be rewritten in terms of potential temperature as:

$$
F_{B}=g \frac{\theta_{\text {parcel }, z+d z}-\theta_{z+d z}}{\theta_{z+d z}}
$$

Noting that $\theta_{\text {parcel }, z+d z}=\theta_{z}$ :

$$
\begin{aligned}
F_{B} & =g\left(\frac{\theta_{z}-\theta_{z+d z}}{\theta_{z+d z}}\right) \\
& =-\frac{g}{\theta} \frac{d \theta}{d z} d z \\
& =-N^{2} d z
\end{aligned}
$$

Brunt-Väisälä frequency ( $N$ ): the frequency at which an air parcel will oscillate if displaced vertically and acted upon by the restoring force arising from the buoyancy of the parcel (also known as the buoyancy frequency).
$N=\left(\frac{g}{\theta} \frac{d \theta}{d z}\right)^{0.5}$
What property of the environment determines the magnitude of the buoyancy force?

What happens to an air parcel that is displaced vertically by a dry adiabatic process if:
$d \theta /\left.d z\right|_{\text {environment }}=0$
$d \theta /\left.d z\right|_{\text {environment }}<0$
$d \theta /\left.d z\right|_{\text {environment }}>0$
It can be shown (see text) that:

$$
\begin{aligned}
\frac{1}{\theta} \frac{d \theta}{d z} & =\frac{1}{T}\left(\frac{d T}{d z}+\frac{g}{c_{p}}\right) \\
& =\frac{1}{T}\left(\Gamma_{d}-\Gamma\right)
\end{aligned}
$$

Which allows $N$ to be expressed in terms of the dry and environmental lapse rates as:

$$
N=\left[\frac{g}{T}\left(\Gamma_{d}-\Gamma\right)\right]^{0.5}
$$

Example: Estimate a typical value of $N$ and the associated period of oscillation $\left(\tau=\frac{2 \pi}{N}\right)$ in the troposphere?

Oscillations of this type can give rise to gravity waves.
Gravity waves can be excited by flow over topography as shown below:


For a flow with wind speed $U$ over a series of ridges separated by distance $L$ the topography will force an atmospheric oscillation with a period given by:

$$
\tau=\frac{L}{U}
$$

Example: Assume that the topography of a region is characterized by ridges that are 10 km apart in the direction of the flow.

If the environmental lapse rate is 5 deg $\mathrm{Ckm}^{-1}$ and the temperature is 20 deg $C$ find the wind speed $(U)$ that would create an orographically driven atmospheric oscillation that matches the period of the buoyancy oscillation.

## Static Stability in Saturated Air

For a saturated air parcel the rate of temperature decrease with height is given by the saturated adiabatic lapse rate $\left(\Gamma_{s}\right)$.

In the situation where an air parcel may be either unsaturated or saturated the stability classes discussed above need to be modified to account for the difference between the dry and saturated adiabatic lapse rate.

What are the static stability classes for an air parcel in an environment with:
$\Gamma>\Gamma_{d}$ ?
$\Gamma=\Gamma$ ?
$\Gamma_{\mathrm{d}}>\Gamma>\Gamma_{\mathrm{s}}$ ?
$\Gamma=\Gamma_{s}$ ?
$\Gamma<\Gamma_{s}$ ?
While it is useful to consider static stability criteria based on comparisons of the environmental lapse rate to the dry and saturated adiabatic lapse rates it is often easier to visualize these stability classes on a skew $T-\log p$ diagram.

Level of free convection (LFC or $z_{\text {LFC }}$ ): The height at which the temperature of a rising air parcel ( $T_{\text {parcel }}$ ) first exceeds the temperature of the environment ( $T_{\text {env }}$ ).

Equilibrium level (EL or $z_{E L}$ ): The height above the LFC at which the temperature of a rising air parcel is once again equal to the temperature of the environment.

Convective available potential energy (CAPE): The vertically integrated air parcel buoyancy between the LFC and the EL

Graphically the CAPE is the area between the air parcel and environmental temperature curves on a skew T diagram between the LFC and EL.

Mathematically, the CAPE can be calculated as:

$$
C A P E=g \int_{z_{L F C}}^{z_{E L}}\left(\frac{T_{\text {parcel }}-T_{e n v}}{T_{e n v}}\right) d z
$$

CAPE is a measure of the energy available to accelerate an air parcel vertically in a thunderstorm, and the maximum vertical velocity that can be achieved in a thunderstorm is given by:

$$
w=(2 \cdot C A P E)^{0.5}
$$

Typical values of CAPE

| 0 to $500 \mathrm{~J} \mathrm{~kg}^{-1}$ | Very weak instability |
| :--- | :--- |
| 500 to $1500 \mathrm{~J} \mathrm{~kg}^{-1}$ | Weak instability |
| 1500 to $2500 \mathrm{~J} \mathrm{~kg}^{-1}$ | Moderate instability |
| 2500 to $4000 \mathrm{~J} \mathrm{~kg}^{-1}$ | Strong instability |
| $>4000 \mathrm{~J} \mathrm{~kg}^{-1}$ | Extreme instability |

What is the maximum vertical velocity for each of these values of CAPE?
Convective Inhibition (CIN): The energy barrier to be surmounted before an air parcel reaches the LFC.

Example: KFWD 5 June 200500 UTC


Example: Indicate the location of the LFC and EL on a skew T diagram for KFWD on 5 June 200500 UTC. Also indicate the areas on this sounding that represent the CAPE and CIN.

